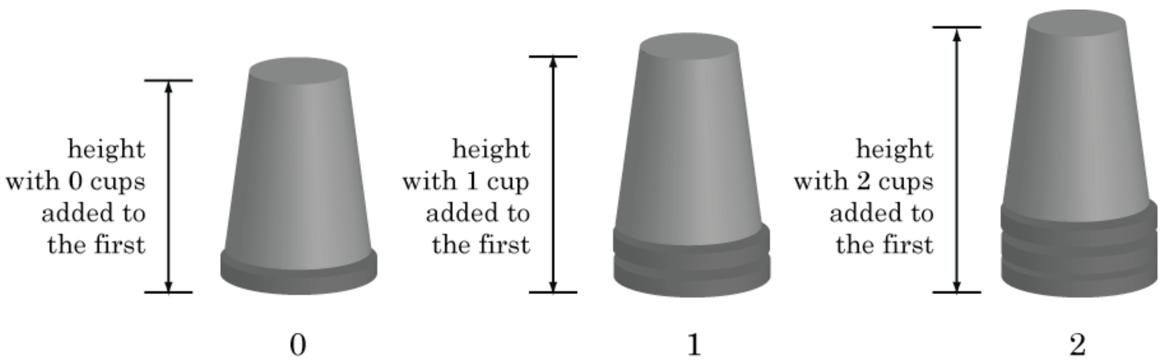
Cup Packing Activity Sheet **Name: _KEY_**

You have been hired by a company that makes disposable drinking cups for a popular coffee retailer. The company typically ships cups in boxes of 100. They need to determine a method of packaging cups that will be least expensive and use the least amount of resources. Your task is to provide this information.

Investigation Steps

(1) Stack cups, one at a time. Record the total height of the cup stack for each number of cups added to the first.

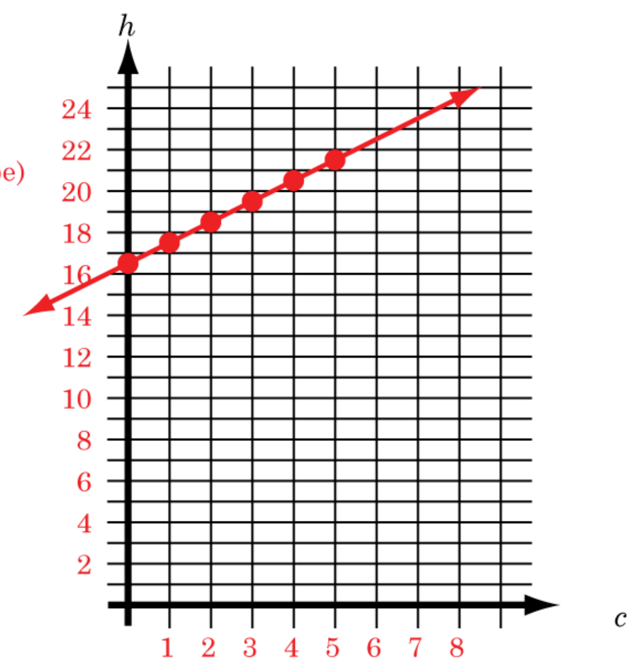


(2) Record your measurement data for the styrofoam cups in the table below. Then make graph of the data set on the coordinate grid below.

Cups added to the first (c)	Height in centimeters (h)
intercept 0	16.2
1	17.3
2	18.4
3	19.5
4	20.6
5	21.7
n	1.1c + 16.2

+1.1 (slope)

slope intercept



(3) Predict how tall a stack of 100 cups would be. How tall would a stack of *n* cups be? Explain how you determined each (hint: use your graph and/or your table of data).

Looking at the table of values, I noticed that each time a new cup was added to the stack, the height of the stack increased by 1.1 cm. This is the slope of the line connecting the data points. The initial height of the stack is 16.2 cm. This corresponds to the *y*-intercept of the line connecting the data points.

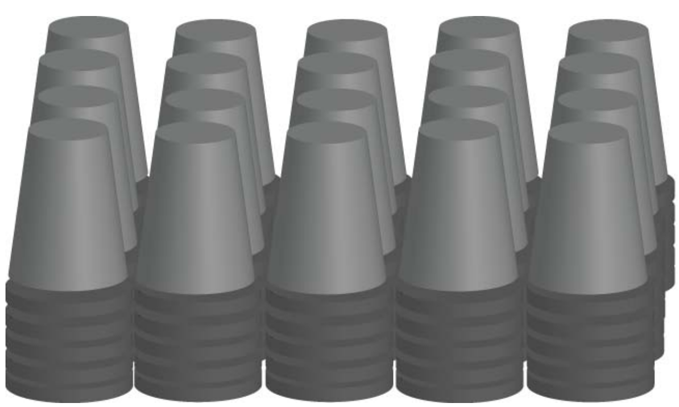
The equation of the line describing the height of the cups (h) with respect to cups added to the first (c) is given by $h = 1.1c + 16.2$. To predict the height of a stack of 100 cups, we can use the formula with c = 99:

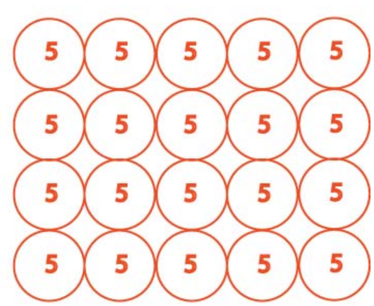
$h = 1.1c + 16.2$
$h = 1.1(99) + 16.2$
$h = 108.9 + 16.2$
$h = 125.1$ cm

(4) You have been asked to determine a method of packaging cups that will be as inexpensive as possible, while using resources as efficiently as possible. On this page and the next, sketch/describe several different configurations of 100 cups. When splitting the cups up into several stacks, each stack must contain the same number of cups.

Configuration 1:

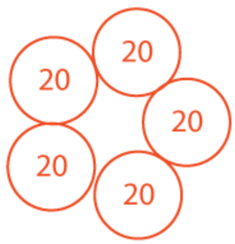
I split the cups into stacks of 5. Twenty of these stacks contains 100 cups. An arrangement of 4 rows and 5 columns of these stacks is shown below. The diagram at the right (below) shows "top view" of 20 individual stacks. The number on each stack shows the number of cups in the stack.





Configuration 2:

We can also divide the cups into 5 stacks of 20. Below is a "top view" of one such arrangement. The cups will be placed into a large "tube" that holds 5 stacks of cups.



Configuration 3:

We can also divide the cups into 4 stacks of 25. Below is a "top view" of one such arrangement.

5. Assume that your cups will be shipped in cardboard containers. For each of the configurations that you described/sketched in item (4), determine (a) the amount of cardboard needed to contain the cups, (b) the total cost of the cardboard (assume that cardboard costs $0.0001 per sq. cm.), and (c) the total volume of the container. Show work below. Place your final calculations in the table on the next page.

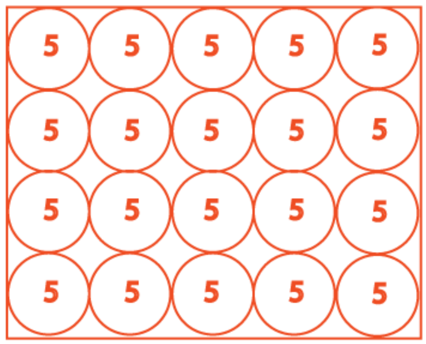
Work (Configuration 1):

Dimensions of Box:

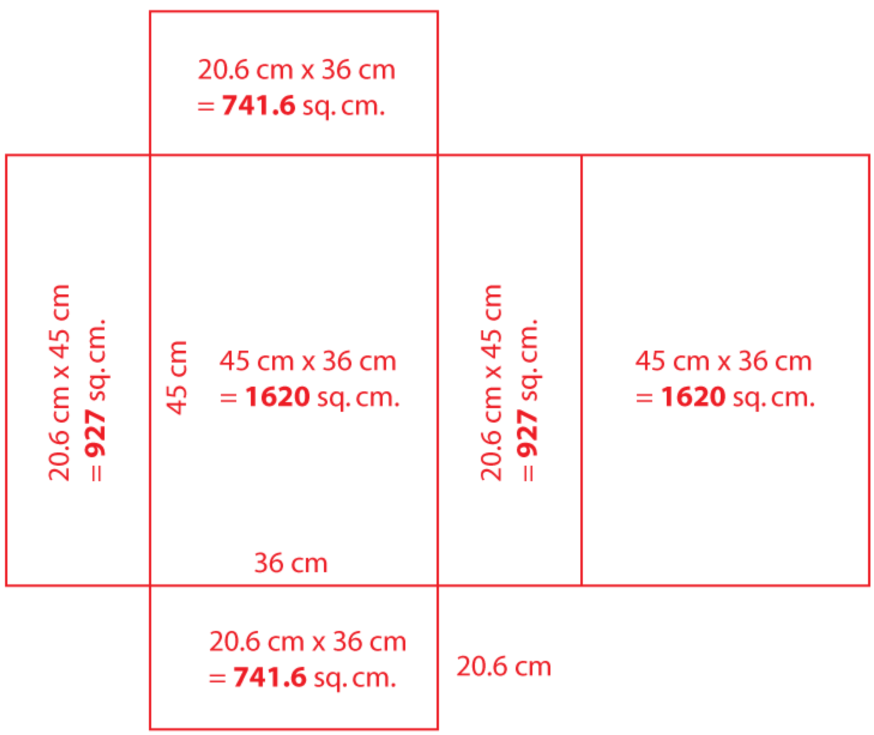
Height of Box: 1.1(4) + 16.2 = 20.6 cm

Width of Box: 9 cm x 5 = 45 cm

Length of Box: 9 cm x 4 = 36 cm



Surface Area of Box (not drawn to scale):



S.A. = 2(**741.6** sq. cm.)+2(**1620** sq. cm.)+2(**927** sq. cm.)

= **6577.2** sq. cm.

Cost = 6577.2 sq. cm. x $0.0001/sq. cm. **≈ $0.66**

Volume = Area of Base • Height

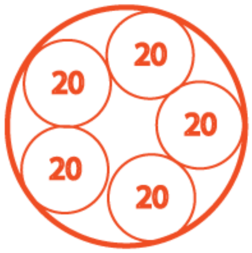
= 45 cm • 36 cm • 20.6 cm

= **33,372 cubic cm**

Work (Configuration 2): Cylinder Option

Dimensions of Cylinder:

Height of Cylinder: 1.1(19) + 16.2 = 37.1 cm



Radius of Cylinder: Approx. 12.16 cm
(Note: Measured with Ruler)

Surface area of cylinder:

Circumference of Base = 2π•radius
≈ 2π•(12.16 cm)
≈ 76.40 cm.

Area of Base = π•radius•radius
≈ π•(12.16 cm)•(12.16 cm)
≈ 464**.53 sq. cm.**

S.A. of Cylinder = Circumference of Base • Height + 2•Area of Base
≈ 76.40 cm • 37.1 cm + 2(464.53 sq. cm.)
≈ 3763.5 sq. cm.

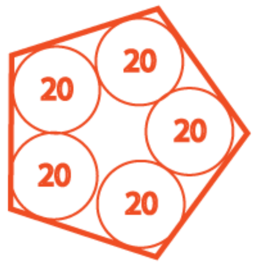
Cost = 3763.5 sq. cm. x $0.0001/sq. cm. **≈ $0.38**

Volume = Area of Base • Height
≈ 464.53 sq. cm. • 37.1 cm
≈ 17234.06 cubic cm.

Work (Configuration 2): Pentagon Option

Dimensions of Pentagonal Prism:

Height of Prism: 1.1(19) + 16.2 = 37.1 cm



Length of Side of Pentagon: Approx. 17.8 cm
Length of Apothem: Approx. 12.2 cm
(Note: Measured with Ruler)

Surface area of cylinder:

Perimeter of Base ≈ 5 • 17.8 cm
≈ 89 cm

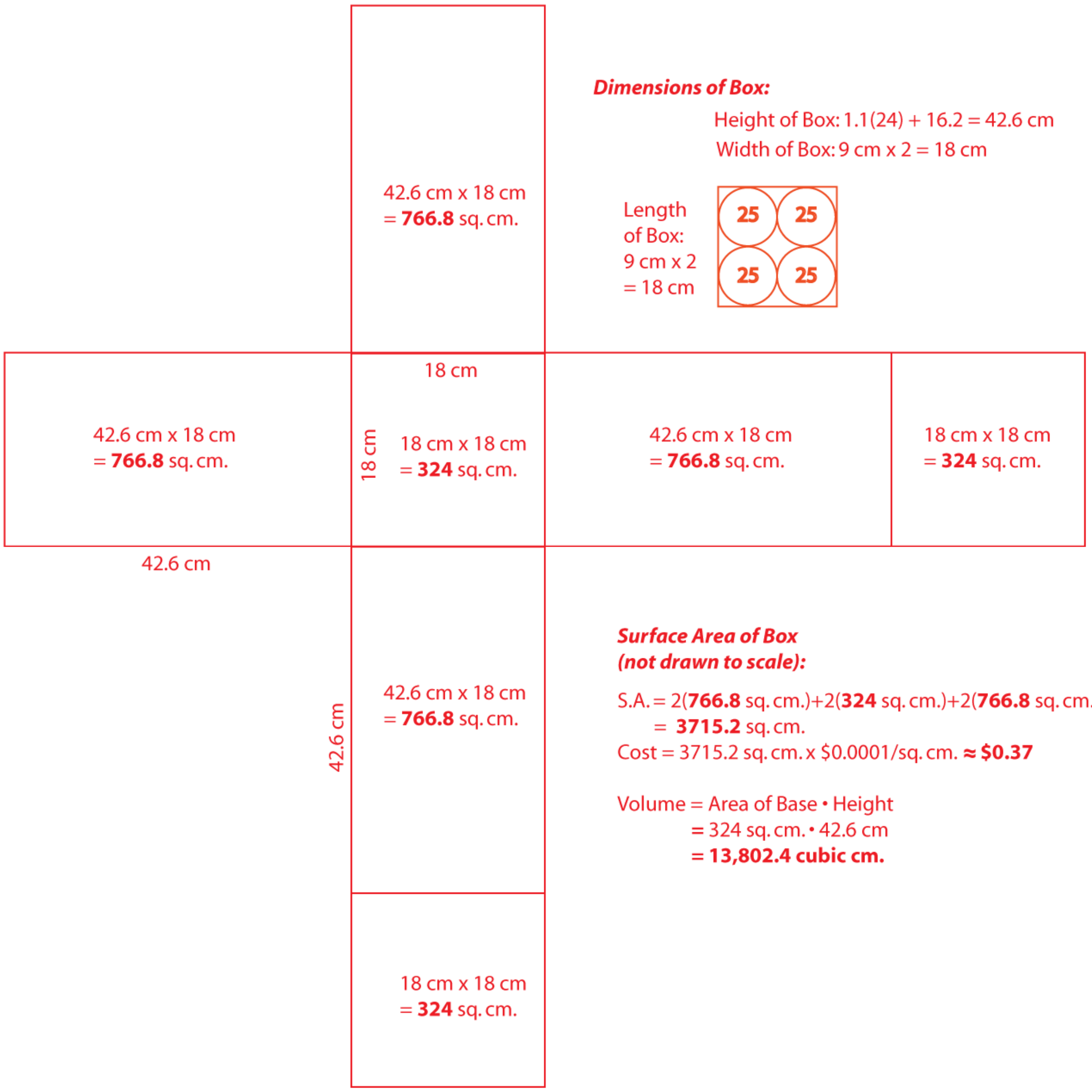
Area of Base = (1/2) • Apothem • Perimeter of Base
≈ (1/2) • 12.2 cm • 89 cm
≈ 542.9 sq. cm.

S.A. of Cylinder = Perimeter of Base • Height + 2 • Area of Base
≈ 89 cm • 37.1 cm + 2(542.9 sq. cm.)
≈ 4387.7 sq. cm.

Cost = 4387.7 sq. cm. x $0.0001/sq. cm. ≈ **$0.44**

Volume = Area of Base • Height
≈ 542.9 sq. cm. • 37.1 cm
≈ 20141.6 cubic cm.

Work (Configuration 3): Prism Option



Dimensions of Box:

Height of Box: 1.1(24) + 16.2 = 42.6 cm
Width of Box: 9 cm x 2 = 18 cm

Length of Box: 9 cm x 2 = 18 cm

Surface Area of Box (not drawn to scale):

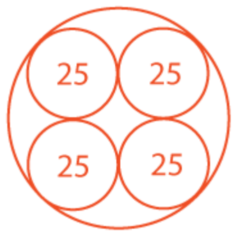
S.A. = 2(**766.8** sq. cm.)+2(**324** sq. cm.)+2(**766.8** sq. cm.)
= **3715.2** sq. cm.
Cost = 3715.2 sq. cm. x $0.0001/sq. cm. ≈ **$0.37**

Volume = Area of Base • Height
= 324 sq. cm. • 42.6 cm
= 13,802.4 cubic cm.

Work (Configuration 3): Cylinder Option

Dimensions of Cylinder:

Height of Cylinder: 1.1(24) + 16.2 = 42.6 cm



Radius of Cylinder: Approx. 10.8 cm
(Note: Measured with Ruler)

Surface area of cylinder:

Circumference of Base = 2π•radius
≈ 2π•(10.8 cm)
≈ 67.9 cm.

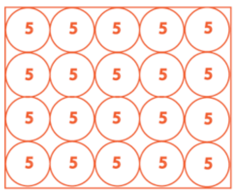
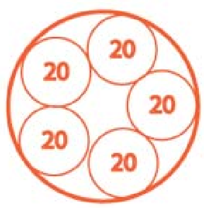
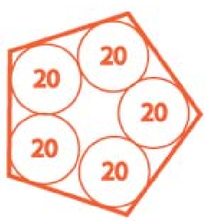
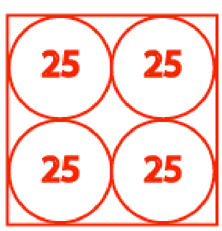
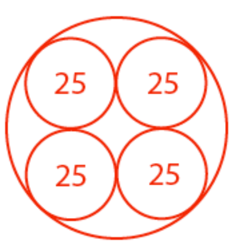
Area of Base = π•radius•radius
≈ π•(10.8 cm)•(10.8 cm)
≈ 366.4 sq. cm.

S.A. of Cylinder = Circumference of Base • Height + 2•Area of Base
≈ 67.9 cm • 42.6 cm + 2(366.4 sq. cm.)
≈ 3625.3 sq. cm.

Cost = 3625.3 sq. cm. x $0.0001/sq. cm. ≈ **$0.36**

Volume = Area of Base • Height
≈ 366.4 sq. cm. • 42.6 cm
≈ 15608.6 cubic cm.

Calculation Results

Cup Configuration	Amount of Cardboard Required (square cm)	Total Cost (Dollars)	Volume of Container (cubic cm)
	6,577.2 square cm	$0.66	33,372 cubic cm
	3,763.5 square cm	$0.38	17,234 cubic cm
	4,387 square cm	$0.44	20,142 cubic cm
	3,715 square cm	$0.37	13,802 cubic cm
	3,625.3 square cm	$0.36	16,609 cubic cm

6. Based on your cost and volume calculations, recommend the inside dimensions of a carton that will be least expensive and use resources as efficiently as possible.

Answers will vary based on those generated by students.