

## Student Council Donut Sale – Solutions

The Student Council at a high school where 2750 students attend sells donuts on Friday mornings as a fund-raiser. Donuts cost \$.20 each wholesale. The student body was surveyed to determine the maximum amount of money students were willing to pay for a donut.

Based on the survey, it was estimated how many donuts would be sold at a particular selling price as listed in the table:

Selling Price (in dollars)	Number of Donuts Sold	Gross Income	Cost of donuts	Net Profit
0	2400	0	480	-480
.10	2160	216	432	-216
.20	1920	384	384	0
.30	1680	504	336	168
.40	1440	576	288	288
.50	1200	600	240	360
.60	960	576	192	384
.70	720	504	144	360
.80	480	384	96	288
.90	240	216	48	168
1.00	0	0	0	0

1. Selling price is the independent variable  $x$ .
  - a) Using the data, plot the following:
    - Selling price versus Number of donuts sold
    - Selling price versus Gross income generated
    - Selling price versus Cost of purchasing donuts sold at that price
    - Selling price versus Net profit
  - b) Draw a “line of best fit” for each.
  - c) Discuss similarities and differences in the graphs.
  - d) Identify patterns in the table and explain how they connect to the graph.
  - e) Write an explicit rule for the number of donuts sold in terms of selling price  $x$ .
 
$$\text{Number of donuts sold} = 2400 - (240/.10) x = 2400 - 2400x$$
  - f) Verbally explain the relationship between selling price and number of donuts sold.
 

*For every \$0.10 price increase, they sell 240 fewer donuts.*

2. a) Identify 2 selling prices that generate the same amount of income. Find 2 more pairs. Locate them on the graph. What do you notice?  
*Answers vary. The selling prices are symmetric on the graph and at equal distances from the maximum.*
- b) Is there any selling price that does not generate the same income as another selling price? If so, locate this price on the graph. What do you notice?  
*(0.50, 600) It is at the vertex, at a maximum.*
- c) Locate and name the coordinates of the  $x$ -intercepts for the graph of income. What is the meaning of these points in the context of the problem?  
*(0,0) and (1.00, 0); (price, 0 income); the prices for which there is no income*
3. a) Write explicit equations for the following:
- Income as a function of selling price:  $I(x) = (\text{selling price})(\text{number sold})$   
 $= (x)(2400 - 2400x) = 2400x - 2400x^2$
  - Cost as a function of selling price:  $C(x) = (0.20)(\text{number sold})$   
 $= (0.20)(2400 - 2400x) = 480 - 480x$
  - Profit as a function of selling price:  $P(x) = I(x) - C(x)$   
 $= (2400x - 2400x^2) - (480 - 480x)$   
 $= -2400x^2 + 2880x - 480$
- b) Discuss similarities and differences in the explicit rules.  
*Cost is linear or 1<sup>st</sup> degree; income and profit are quadratic or 2<sup>nd</sup> degree*
4. Use the graphs to come up with a selling plan to maximize profits. Include in your plan:
- a) A selling price \$0.60 (*most profit*), not \$0.50 (*most income*)
  - b) Maximum income and profit that can be generated \$600; \$384
  - c) The break-even point (where income and costs are equal) *Selling prices of \$0.20 and \$1*
  - d) Selling prices where costs exceed income  $x < \$0.20$
5. Algebraically find:
- a) The break-even points for income and cost  
 $2400x - 2400x^2 = 480 - 480x$   
 $x = 1$  or  $0.20$
  - b) The break-even points for profit and cost  
 $-2400x^2 + 2880x - 480 = 480 - 480x$   
 $x = 1$  or  $0.40$