**The Graphs of the “Absolute Value” Function with Graphing Calculators**
*by S. Asli Ozgun-Koca, PhD.*

Subject: The Graphs of the “Absolute Value” Function
Grade Level: 6–8
Time: 3 consecutive days, 45 minutes per day
Materials:

- one TI-92 or TI-89 for every pair of students
- one worksheet for each student

**Alignment with Ohio Academic Content Standards**

Patterns, Functions and Algebra Standard

Grades 5–7

- A. Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.
- B. Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.

Grades 8–10

- C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.
- E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.

Mathematical Processes Standard

Grades 5–7

- F. Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
- H. Use representations to organize and communicate mathematical thinking and problem solutions.
- I. Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem.

Grades 8–10

- C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.
- D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

- E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

**Alignment with NCTM Standards**

Algebra Standard
Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols.

Reasoning and Proof Standard
Instructional programs from prekindergarten through grade 12 should enable all students to—

- make and investigate mathematical conjectures.

Representation Standard
Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems.

**Rationale:** The “absolute value” function is one of the best examples in mathematics curriculum where subjects become more abstract and harder to interpret for younger students. However, the real-life examples could help students construct a meaning for the absolute value. For instance, when we talk about our weight, we say that “I gained 3 pounds” or “I lost 3 pounds” instead of “I gained +3 pounds” or “I lost -3 pounds.” Another real-life example of the absolute value function could be the temperature. The weatherman says, “Today, it will be 3 degrees hotter [or colder] than yesterday,” rather than “It will be +3 degrees hotter than yesterday” or “It will be -3 degrees colder than yesterday.” These examples may help students to interpret the textbook definition of the absolute value, which is “the distance from zero.” But this construction could get complicated for the students when we add one more variable and carry the absolute value on the coordinate system. Now, technology could be invaluable to visualize the graphs of the absolute value function and make the underlying concept more concrete. By providing multiple representations on the same screen, TI-92s and TI-89s can help students make conjectures about the behaviors of the graphs of absolute value functions and then test their conjectures.

**Background:** *Using Handheld Graphing Technology in Secondary Mathematics: What Scientifically Based Research Has to Say* (2003) by Texas Instruments concluded that "handheld graphing technology can have a positive impact on student learning in a range of settings and using a variety of instructional approaches. In particular, the research shows that use of graphing handhelds can have a positive impact both on general skill and understanding of algebra concepts and, more specifically, on student comprehension of functions." (p. 14; http://education.ti.com/sites/US/downloads/pdf/whitepaper.pdf) Burrill et al. (2002) presents more detailed results from the same meta-analysis. Dunham and Dick (1994) presents an overview of some results of research on the use of graphing calculators in mathematics education, especially the effects of graphing calculators on students' achievement, conceptual understanding, problem solving, and classroom dynamics.

There are papers and lesson plans on absolute value function in the mathematics education literature (Arcidiacono, 1983; Brumfiel, 1980; Horak, 1994; National Council of Teachers of Mathematics [NCTM], 1991; Parish, 1992; Stallings-Roberts, 1991). NCTM described a classroom vignette about a teacher, Ms. Chavez, who encouraged her students to sketch the graphs of absolute value functions in the form of $y = |x| \pm c$. Then students were asked to compare their graph of $y = |x| \pm c$ with the graph of $y = |x|$ and to write a paragraph about it. The class used computers to check their graphs during the class discussion. Horak used graphing calculators to help students visualize the solutions of absolute value equations by highlighting the graphical interpretation of the solution.

The aim of this lesson plan is to use handheld graphing technology and multiple representations to help students visualize the graphs of absolute value functions and make conjectures about the relationships between the algebraic and graphical forms of absolute value functions.

## The Graphs of the “Absolute Value” Function with Graphing Calculators

### TEACHER WORKSHEET

*The aim of this lesson plan is to use handheld graphing technology and multiple representations to help students visualize the graphs of absolute value functions and make conjectures about the relationships between the algebraic and graphical forms of absolute value functions.*

**Day 1**

*A discussion about the application of the concept of absolute value in daily life (mentioned in the Rationale Section) could help students become interested in the lesson.*

1. What do you understand from the concept of “absolute value”?

2. What are the effects of absolute value function on any number or function?

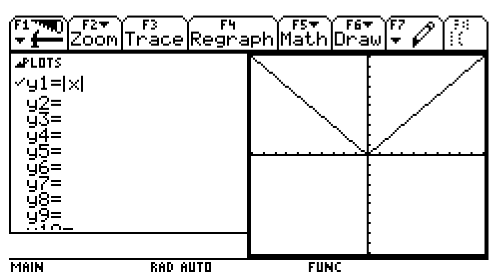
3. Are there any differences between the graph of a function and its graph after absolute value function was applied?

*These three questions are designed to determine students’ existing knowledge on absolute value and the graphs of absolute value functions.*

*Organize students in pairs for engagement and discussion. Distribute one graphing calculator to each pair of students.*

*Make sure that students use the split-screen mode for this activity (half equation-view and half graph-view by using the Mode button) in order to help them associate the algebraic and the graphical representations of the absolute value function. The graph of y = /x/ is drawn on the screen, and this graph is kept on the screen with the aim of further comparisons during the entire lesson.*

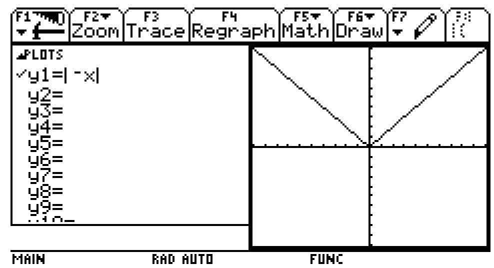
4. Guess the graph of $y = |-x|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side. [*It is always beneficial to have students predict before they see what technology produces in order to strengthen and validate their mathematical thinking.*]



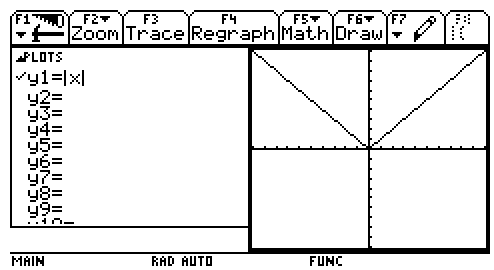
*During the lesson, teacher could also use an overhead transparency/LCD display for students to draw their predictions on the coordinate system where y = $|x|$ was drawn, and also to watch the graphing calculator draw the graphs and compare them during the class discussions.*

*The screenshots after the graphs have been drawn are excluded on the worksheets for students.*

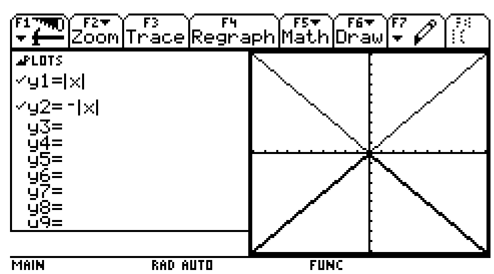
5. Draw the graph of y = $|-x|$. What are the differences between the graphs of y = $|-x|$ and y = $|x|$? *[Interpreting what technology produced helps students re-think their predictions.]*




6. Guess the graph of y = $-|x|$ by drawing on the same coordinate system where y = $|x|$ was drawn on the right-hand side. *[The goal is to help students reconstruct or strengthen the concept of absolute value.]*




7. Draw the graph of y = $-|x|$, using a different style *[choose F6 after highlighting the function*] and without deleting the graph of y = $|x|$. Compare these two graphs. *[At this point, a discussion about the comparison of y = $|-x|$ and y = $-|x|$ might help students apply meaning to the concept of absolute value.]*

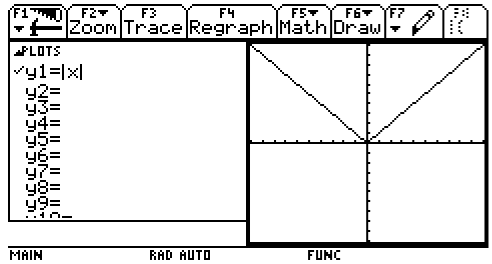



**Day 2**

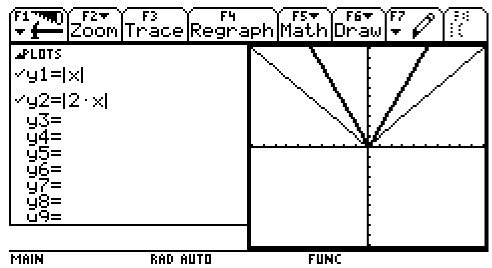
1. Please explain yesterday's lesson for your class-partner as though they missed the class. Try to be as explicit as possible. *[In order to explain the previous day's class to a classmate, students first need to reflect on their thinking and communicate it through writing. This might help students organize their newly constructed concepts.]*

*The common goal of the following questions is to help students construct the effects of the coefficient of x.*

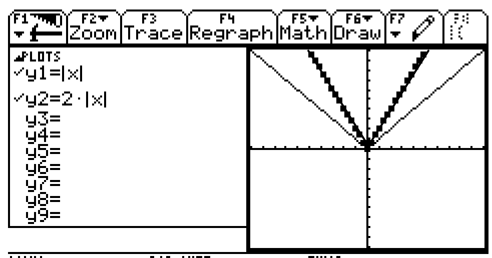
2. Guess the graph of $y = |2x|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side. *[The goal is to help students reconstruct or strengthen the effects of the coefficient of x. If suitable, a discussion about slope of linear relationships could be valuable.]*



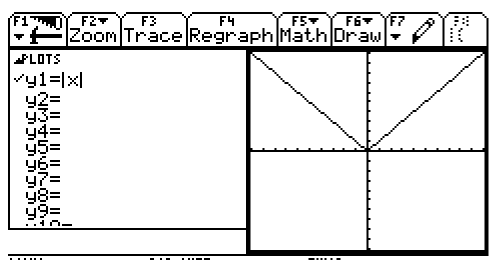

3. Draw the graph of $y = |2x|$ in a different style and without deleting the graph of $y = |x|$. Compare these two graphs.



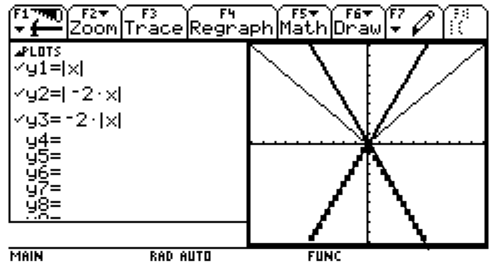

4. Draw the graphs of functions $y = |x|$ and $y = 2|x|$ in different styles on the same coordinate system. Compare them. *[At this point, a discussion about comparing $y = |2x|$ and $y = 2|x|$ might help students re-think the concept of absolute value when the coefficient of x is positive.]*



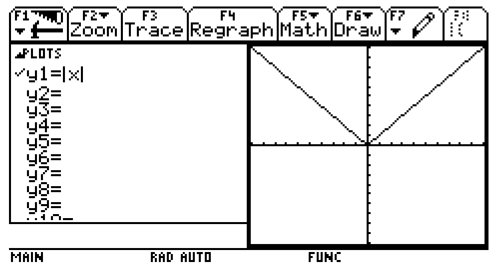

5. Guess the graphs of $y = |-2x|$ and $y = -2|x|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side. *[Now students need to take into account that the coefficient of x is negative, and reflect again on the concept of absolute value and the effects of the coefficient of x at the same time.]*



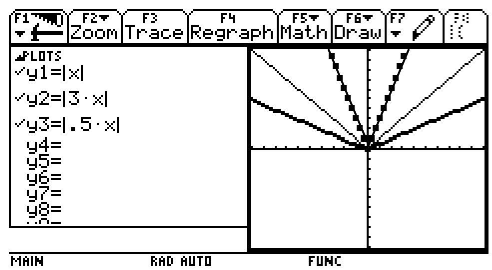
6. Draw the graphs of functions $y = |x|$, $y = |-2x|$, and $y = -2|x|$ in different styles on the same coordinate system. Compare them.




7. Guess the graphs of $y = |3x|$ and $y = |0.5x|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side. *[This exercise is designed to help students think about the effects of the magnitude of the coefficient of x.]*




8. Draw the graphs of functions $y = |3x|$ and $y = |0.5x|$ in different styles on one coordinate system. Compare them.



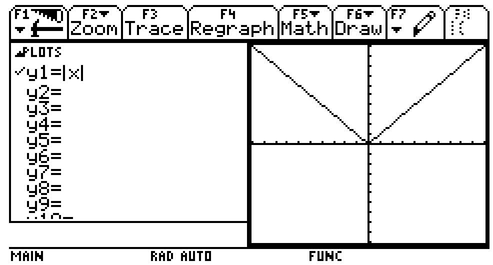

*If needed, students could be asked to draw and compare the graphs of absolute value functions in the form of $y = \pm|ax|$ where "a" could be positive or negative with various magnitudes.*

**Day 3**

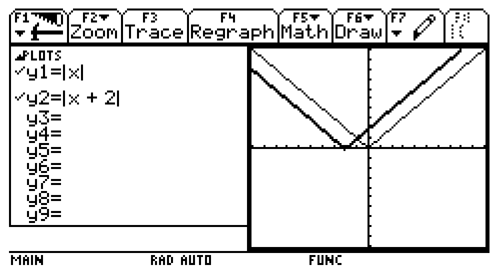
1. Please summarize the lessons that we completed in the last two days in your own words. *[First, students will need to reflect on what they have been doing during the previous two days. This exercise is designed to help students organize their newly constructed concepts.]*

*On Day 3, a process similar to Day 2's is repeated for $y = |x \pm a|$ and $y = |x| \pm a$.*

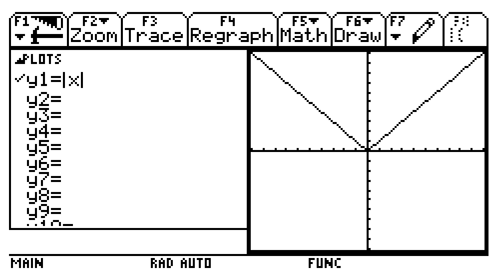
2. Guess the graph of $y = |x + 2|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side. *[This exercise is designed to help students think about the effects of the added components in the absolute function, which will result in a horizontal shift. This activity could be an introduction or connection to the concept of mathematical translations.]*



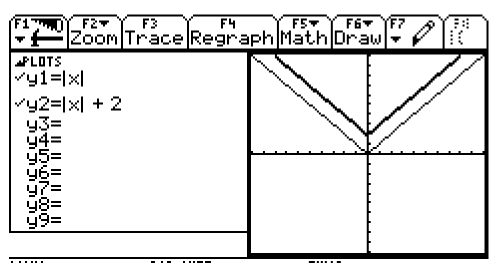

3. Draw the graph of $y = |x + 2|$ in a different style and without deleting the graph of $y = |x|$. Compare these two graphs.



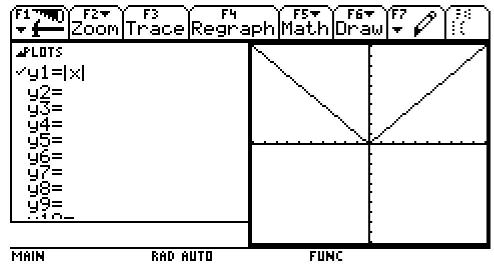

4. Guess the graph of $y = |x| + 2$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side. *[The goal is now to help students think about the difference between $y = |x| + a$ and $y = |x + a|$. The goal is to create an environment for students to study the effects of the added components outside of the absolute function, which will result in a vertical shift.]*



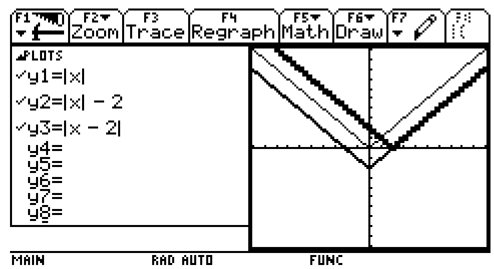

5. Draw the graph of $y = |x| + 2$ in a different style and without deleting the graph of $y = |x|$. Compare these two graphs.



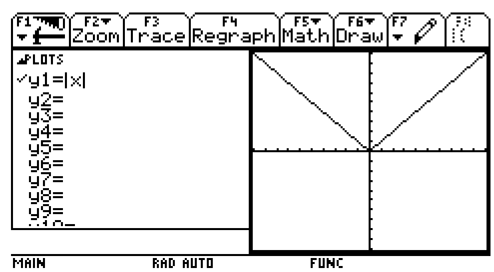
6. Guess the graphs of y = $|x|$ - 2 and y = $|x - 2|$ by drawing on the same coordinate system where y = $|x|$ was drawn on the right-hand side. *[Now the goal is to help students think about the role of the sign of "a" in the forms of y = $|x + a|$ and y = $|x|$ + a. Students should reflect on what causes the graph to move to the right as opposed to the left, or up as opposed to down.]*




7. Draw the graphs of functions y = $|x|$, y = $|x|$ - 2, and y = $|x - 2|$ in different styles on the same coordinate system. Compare them.




8. Guess the graphs of y = $|x|$ + 4 and y = $|x - 5|$ by drawing on the same coordinate system where y = $|x|$ was drawn on the right-hand side. *[This exercise is designed to help students think about the effects of the magnitude of added or subtracted components.]*




9. Draw the graphs of functions y = $|x|$, y = $|x|$ + 4, and y = $|x – 5|$ in different styles on the same coordinate system. Compare them.

**The Graphs of the "Absolute Value" Function with Graphing Calculators**

**STUDENT WORKSHEET**

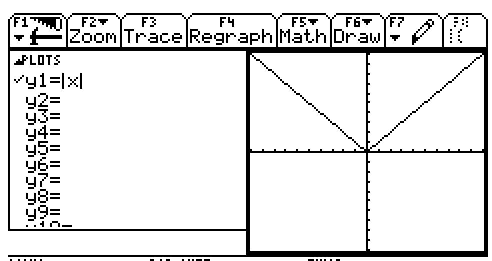
**Day 1**

1. What do you know about the concept of "absolute value"?

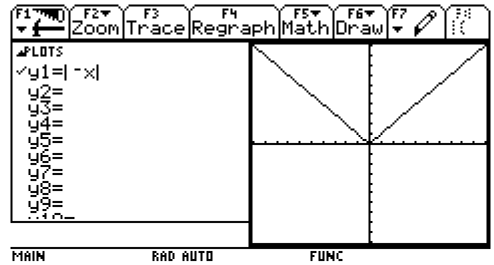
2. What are the effects of the absolute value function on any number or function?

3. Are there any differences between the graph of a function and its graph after the absolute value function was applied?

4. Guess the graph of $y = |-x|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side.



5. Draw the graph of y = $|-x|$. What are the differences between the graphs of y = $|-x|$ and y = $|x|$?

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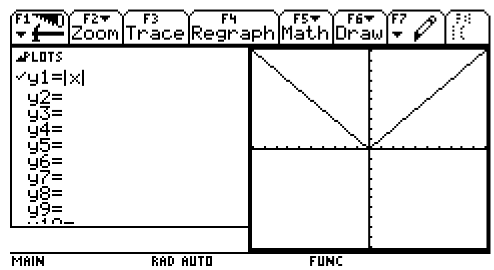



7. Draw the graph of y = $-|x|$ in a different style and without deleting the graph of y = $|x|$. Compare these two graphs.

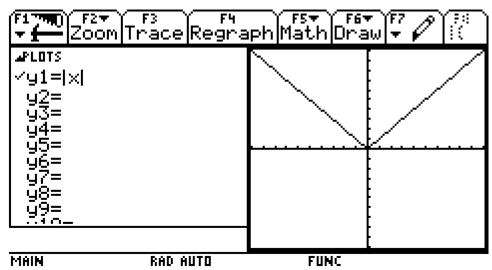
**The Graphs of the "Absolute Value" Function with Graphing Calculators**

**STUDENT WORKSHEET**

**Day 2**

1. Please explain yesterday's lesson for your class-partner as though they had missed the class. Try to be as explicit as possible.

2. Guess the graph of y = $|2x|$ by drawing on the same coordinate system where y = $|x|$ was drawn on the right-hand side.

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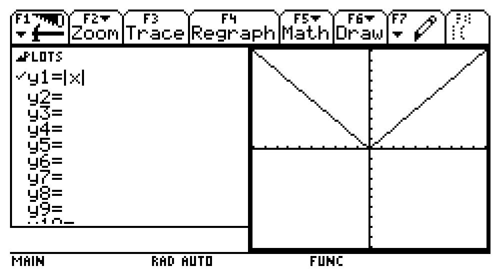
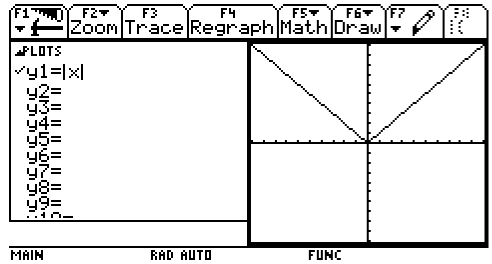
4. Draw the graph of functions $y = |x|$ and $y = 2|x|$ in different styles on the same coordinate system. Compare them.

5. Guess the graph of $y = |-2x|$ and $y = -2|x|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side.




6. Draw the graph of functions $y = |x|$, $y = |-2x|$, and $y = -2|x|$ in different styles on the same coordinate system. Compare them.

7. Guess the graphs of y = $|3x|$ and y = $|0.5x|$ by drawing on the same coordinate system where y = $|x|$ was drawn on the right-hand side.

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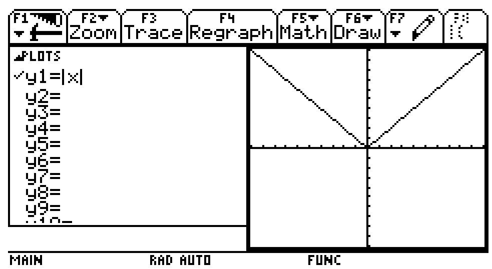
**The Graphs of the "Absolute Value" Function with Graphing Calculators**

**STUDENT WORKSHEET**

**Day 3**

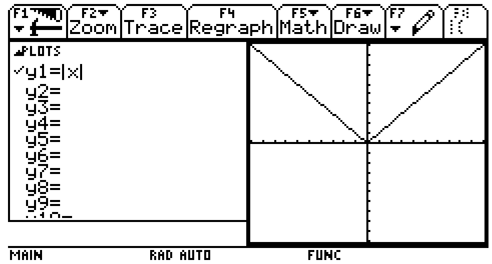
1. Please summarize the lessons from the last two days in your own words.

2. Guess the graph of $y = |x + 2|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side.



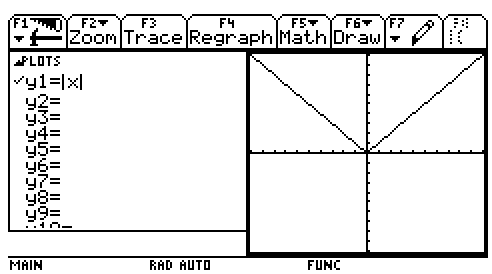

3. Draw the graph of $y = |x + 2|$ in a different style and without deleting the graph of $y = |x|$. Compare these two graphs.

4. Guess the graph of $y = |x| + 2$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side.



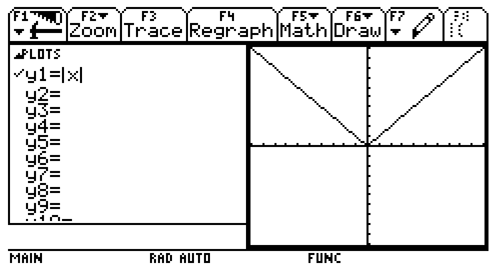

5. Draw the graph of $y = |x| + 2$ in a different style and without deleting the graph of $y = |x|$. Compare these two graphs.

6. Guess the graphs of $y = |x| - 2$ and $y = |x - 2|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side.



7. Draw the graph of functions $y = |x|$, $y = |x| - 2$, and $y = |x - 2|$ in different styles on the same coordinate system. Compare them.

8. Guess the graphs of $y = |x| + 4$ and $y = |x - 5|$ by drawing on the same coordinate system where $y = |x|$ was drawn on the right-hand side.



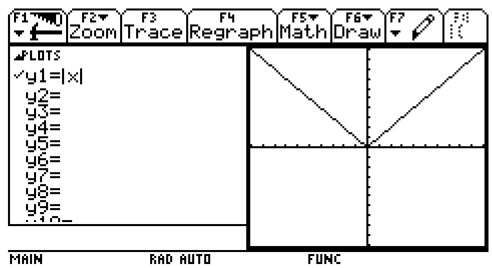

9. Draw the graphs of functions $y = |x|$, $y = |x| + 4$, and $y = |x - 5|$ in different styles on the same coordinate system. Compare them.

## Suggestions for Assessment (Teacher)

*First students could be asked to complete the following tasks in their own time as a homework or in-class assignment. After the assignment has been turned in, class discussion could take place. As during lessons, during the assessment process, the teacher could provide a transparency/LCD display (of the coordinate system where y = $|x|$ was drawn) for students to draw their graph predictions, then to watch the graphing calculator draw the graphs. Compare the graphs as class discussion.*

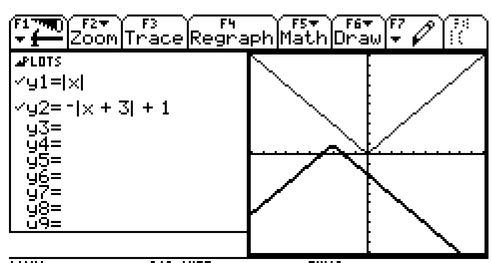
1. a) Guess the graph of $y = -|x + 3| + 1$ by drawing on the same coordinate system where $y = |x|$ was drawn (below). Explain your graph and the reasoning behind it.

*[Students are asked to explain their reasoning in this multi-step problem for reflection purposes. This may also help the teacher identify students' strong and weak conceptions.]*

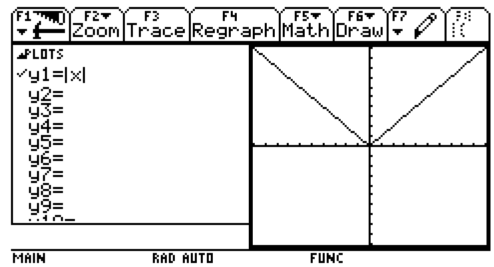



b) Draw the graph of functions $y = |x|$ and $y = -|x + 3| + 1$ in different styles. Compare them. Was your prediction close? Discuss the similarities and differences between your predicted and calculator-produced graphs.

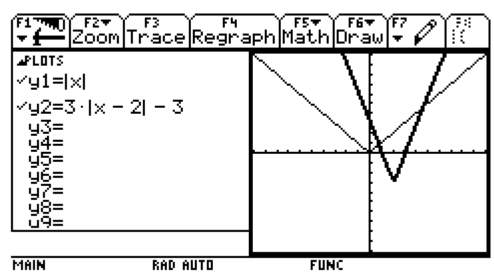
*[Asking students to compare their graphs with calculator-produced graph might help students reflect on their mistakes and also help the teacher detect students' misconceptions.]*



2. a) Guess the graph of $y = 3|x - 2| - 3$ by drawing on the same coordinate system where $y = |x|$ was drawn (below). Explain your graph and the reasoning behind it.



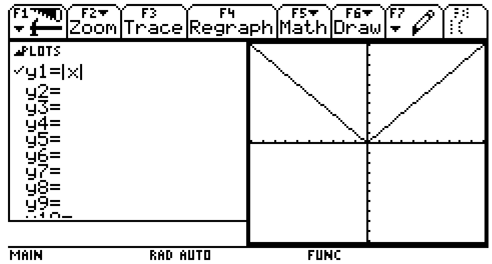

b) Draw the graph of functions $y = |x|$ and $y = 3|x - 2| - 3$ in different styles. Compare them. Was your prediction close? Discuss the similarities and differences between your predictions and the calculator-produced graphs.



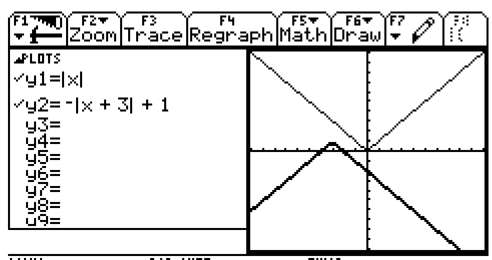

*Similarly, more complex examples and comparisons could be asked for assessment purposes, which would reveal students' understandings and any misinterpretations. This could be done with or without technology, based on the teacher's objectives.*

**Assignment**

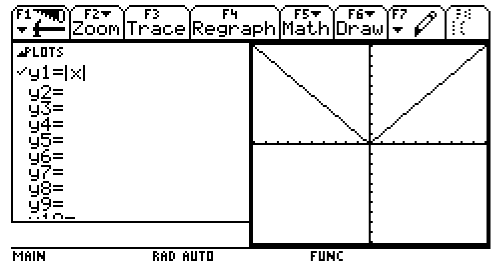
1. a) Guess the graph of $y = -|x + 3| + 1$ by drawing on the same coordinate system where $y = |x|$ was drawn (below). Explain your graph and the reasoning behind it.



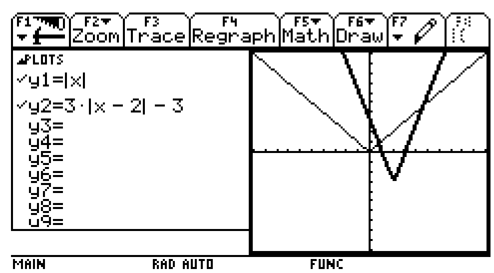

b) Draw the graphs of $y = |x|$ and $y = -|x + 3| + 1$ in different styles on the same coordinate system. Compare them. Was your prediction close? Discuss the similarities and differences between your predictions and calculator-produced graphs.



2. a) Guess the graph of $y = 3|x - 2| - 3$ by drawing on the same coordinate system where $y = |x|$ was drawn (below). Explain your graph and the reasoning behind it




b) Draw the graphs of functions $y = |x|$ and $y = 3|x - 2| - 3$ in different styles on the same coordinate system. Compare them. Was your prediction close? Discuss the similarities and differences between your predictions and calculator-produced graphs.



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