Ohio's Systemic Initiative
Discovery

Area and Volume in 3D—Can't Have One Without the Other!

One-Page Overview

By Robert B. Brown, The Ohio State University

Topics:

Area, Volume

Levels:

Grades 5 – 8

Problem:

Every three-dimensional solid has both a volume and a surface area. This activity explores the relationships between the two.

Getting Started:

Break the class into groups. Give each group 30 to 40 wooden cubes of the same size and a supply of colored stickums. Ask each group to make a 2x2x2 cube. How many wooden cubes does it take? Ask them to paint with stickums, one to a face, each exposed face of each wooden cube. How many stickums does it take? Remove the stickums and reassemble the same eight wooden cubes into another shape. Again paint the exposed faces of each wooden cube with stickums. How many does it take? Compare the results from the different groups.

Ohio Academic Content Standards, 2002

5-7		8-10		11-12	
1. Number, Number Sense and Operations		1. Number, Number Sense and Operations		1. Number, Number Sense and Operations	
2. Measurement	X	2. Measurement	X	2. Measurement	
3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	
4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra	
5. Data Analysis and Probability		5. Data Analysis and Probability		5. Data Analysis and Probability	
Mathematical Processes Problem Solving		**Mathematical Processes** Problem Solving		**Mathematical Processes**	

NCTM Principles and Standards, 2000

6-8		9-12	
1. Number and Operations		1. Number and Operations	
2. Algebra		2. Algebra	
3. Geometry	X	3. Geometry	
4. Measurement	X	4. Measurement	
5. Data Analysis and Probability		5. Data Analysis and Probability	
6. Problem Solving	x	6. Problem Solving	
7. Reasoning and Proof		7. Reasoning and Proof	
8. Communication		8. Communication	
9. Connections		9. Connections	
10.Representation		10.Representation	

Note: Capital X denotes major emphasis; lower case x denotes minor emphasis.

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Topics: Area, Volume	
Levels: Grades 5 - 8	*Timing:* One hour
Materials: Wooden or plastic cubes, 30-40 to each group of students.◎Stickums that will adhere to the faces of the cubes	*Prerequisites:* Building three-dimensional shapes out of cubes

Problem:

Every three-dimensional solid has both a volume and a surface area. This activity explores the relationships between the two.

Goals:

- Develop intuition about three-dimensional shapes
- See how to find the surface area and volume of shapes made from unit cubes.
- See that the surface area and the volume have only a loose relationship-neither determines the other.

Procedure:

1. Break the class into groups. Give each group 30 to 40 wooden cubes of the same size and a supply of colored stickums. Ask each group to make a 2 x 2 x 2 cube. How many wooden cubes does it take?
2. Ask them to paint with stickums, one to a face, each exposed face of each wooden cube. How many stickums does it take?
3. Remove the stickums and reassemble the same eight wooden cubes into another shape. Again paint the exposed faces of each wooden cube with stickums. How many does it take?

4. Compare the results from the different groups. For each different shape list the number of cubes (8), which represents the volume of the shape, vs. the number of painted faces, which represents the surface area of the shape. Express as a ratio the surface area to the volume.
5. Take 27 of the unit cubes and make a 3 x 3 x 3 cube. Paint the outside surface with stickums. How many unit cubes get more than three painted faces? Exactly three painted faces, just two painted faces, just one painted face, no painted faces? What is the volume and the surface area of the 3 x 3 x 3 cube?
6. Remove the stickums and reassemble the same twenty-seven cubes into another shape? Now how many stickums does it take to paint the surface?
7. Compare the results from the different groups.
8. For each different shape, list the number of cubes (27), which represents the volume of the shape, vs. the number of painted faces, which represents the surface area of the shape. Express as a ratio the surface area to the volume.
9. Discuss whether different shapes with the same volume have the same surface area.
10. Discuss whether different shapes with the same surface area have the same volume.
11. Compare the results for the 2 x 2 x 2 cube and the 3 x 3 x 3 cube. What would the students expect for a 4 x 4 x 4 cube? As the size of the cube goes up, what happens to the ratio of the surface area to the volume?

Extensions:

As long as the shapes have to be made from unit cubes, you get the smallest possible surface area for a given volume by making the shape itself as close as possible to a cube. Then if you want the largest possible surface for a given volume, you could allow yourself to break the volume into individual unit cubes and scatter them about. Then each cube, having no contact with any of the others, would get all of its six faces painted, and you'd have the largest possible surface area.

On the other hand, if you are allowed to smooth out the surface of the volume, as if it were made of playdough, then you would get the smallest surface area for a given volume by using a spherical shape. The reasoning as to why this is so is not at all elementary. But if you use playdough of a given volume, you can get as large a surface as you please simply by rolling out the playdough into a snake shape and making the snake longer and more slender.

You could use this activity as a lead-in to the formulas for the volume and surface area of a cube. If a side of the cube has length n, then $V = n^3$ and $S = 6n^2$.

The Mathematics:

Here are some examples of shapes that can be made from eight unit cubes.

Shape	Volume (V)	Surface Area (S)	Ratio V:S
2 x 2 x 2 cube	8	24	8:24
2 x 4 x 1 flat	8	28	8:28
3 x 3 x 1 square with a hole in the middle	8	32	8:32
8 x 1 x 1 long	8	34	8:34

You see that the surface area can change, even though the volume doesn't. Also, the longer and narrower the shape, the larger is its surface area.
Here are some examples of shapes that can be made from 27 unit cubes.

Shape	Volume (V)	Surface Area (S)	Ratio V:S
3 x 3 x 3 cube	27	54	27:54
3 x 6 x 1 flat with a 3 x 3 x 1 flat stacked on it	27	66	27:66
3 x 9 x 1 flat	27	78	27:78
27 x 1 x 1 long	27	110	27:110

You see that the surface area can change, even though the volume doesn't.
With these numbers of cubes there aren't two shapes with the same surface area but different volumes. But if you use twelve cubes stacked as a 3 x 2 x 2 brick, the surface area is 32, the same as one of the shapes with volume 8.

Here is a table showing the volumes and surface areas of cubes of some different sizes.

Size	Volume (V)	Surface Area (S)	Ratio V:S
2 x 2 x 2	8	24	8:24 = 1:3
3 x 3 x 3	27	54	27:54 = 1:2
4 x 4 x 4	64	96	64:96 = 2:3
5 x 5 x 5	125	150	125:150 = 5:6
6 x 6 x 6	216	216	216:216 = 1:1
7 x 7 x 7	343	294	343:294 =7:6

As the size of similar shapes goes up, the ratio of volume to surface area goes up, too.

Relationships to the Ohio Academic Content Standards, 2002:

Grades 5-7:

Measurement Standard
The student will be able to…

- Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.
- Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.
- Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.

Geometry and Spatial Sense Standard
The student will be able to…

- Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.
- Use proportions to express relationships among corresponding parts of similar figures.

Mathematical Processes Standard
The student will be able to…

- Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations

Grades 8-10:

Measurement Standard
The student will be able to…

- Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.
- Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.

Geometry and Spatial Sense Standard
The student will be able to…

- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.
- Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.

Mathematical Processes Standard

The student will be able to…

- Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.

Relationships to the NCTM Principles and Standards, 2000:

Grades 6-8:

Geometry Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Build new mathematical knowledge through problem solving.