How Much Grout?

One-Page Overview

By Robert B. Brown, The Ohio State University

Topics:

Problem solving, Area

Levels:

Grades 5 – 8

Problem:

Suppose that you have some square ceramic tiles that you will lay for a floor. When you put them down, you will leave a little space between them to fill with grout. You will also put a border of grout around the edge of the floor. How much grout does it take?

Getting Started:

Ask the students to imagine that the tiles they have are 4 inches on a side. Have them lay down 48 tiles to make a floor, spacing the tiles 1/4 inch apart. Then they are to imagine that the space between the tiles is filled in with grout and that they must put a border of grout that is 1/4 inch wide around the edge of the floor. The question then is what is the area of the grout? If you tell them how thick the grout is to be, you can ask what volume of grout is needed. Since the volume is proportional to the thickness, the problem can be viewed as an area problem.

Ohio Academic Content Standards, 2002

5-7		8-10		11-12	
1. Number, Number Sense and Operations	X	1. Number, Number Sense and Operations		1. Number, Number Sense and Operations	
2. Measurement	X	2. Measurement	X	2. Measurement	
3. Geometry and Spatial Sense	x	3. Geometry and Spatial Sense	x	3. Geometry and Spatial Sense	
4. Patterns, Functions and Algebra	X	4. Patterns, Functions and Algebra	X	4. Patterns, Functions and Algebra	
5. Data Analysis and Probability		5. Data Analysis and Probability		5. Data Analysis and Probability	
Mathematical Processes Problem Solving Connections		**Mathematical Processes** Problem Solving Connections		**Mathematical Processes**	

NCTM Principles and Standards, 2000

6-8		9-12	
1. Number and Operations	X	1. Number and Operations	
2. Algebra	X	2. Algebra	
3. Geometry	x	3. Geometry	
4. Measurement	X	4. Measurement	
5. Data Analysis and Probability		5. Data Analysis and Probability	
6. Problem Solving	X	6. Problem Solving	
7. Reasoning and Proof		7. Reasoning and Proof	
8. Communication		8. Communication	
9. Connections	X	9. Connections	
10.Representation		10.Representation	

Note: Capital X denotes major emphasis; lower case x denotes minor emphasis.

How Much Grout?

By Robert B. Brown, The Ohio State University

Topics: Problem solving, Area	
Levels: Grades 5 – 8	*Timing:* 1–2 class periods
Materials: Squares, all the same size and made of any stiff material	*Prerequisites:* Operating with fractions Factoring Work with area and perimeter Work with variables

Problem:

Suppose that you have some square ceramic tiles that you will lay for a floor. When you put them down, you will leave a little space between them to fill with grout. You will also put a border of grout around the edge of the floor. How much grout does it take?

Goals:

- Develop skills in problem solving
- Gain familiarity with calculating areas of rectangles
- Practice factoring whole numbers into different pairs of factors
- See how substituting letters for numbers can aid the analysis of a problem
- Examine the relationship between area and perimeter by discovering how the perimeter of the floor affects the amount of grout needed

Big Ideas:

- Calculating area
- Factors
- Variables
- Relationship between area and perimeter

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Procedure:

1. Ask the students to imagine that they have tiles that are 4 inches on a side. Have them draw 48 tiles to make a floor, spacing the tiles 1/4 inch apart. They are to imagine that the space between the tiles is filled with grout and that they must put a border of grout 1/4 inch wide around the edge of the floor. The question is: what is the area of the grout? If you tell them how thick the grout is to be, you can ask what volume of grout is needed as well.

2. By starting with 48 tiles, which has numerous factors but is not a perfect square, each student or group of students may choose a different shape for the floor. Students who are striving for an all-purpose shape will most likely arrange the tiles in a 6 x 8 rectangle. Others may pick a 4 x 12 array, and some may even propose a 1 x 48 shape. With this large a number of tiles the possibilities are numerous, especially when you take into account that the floors need not be rectangular and that they may even have holes within (for support columns, if you like). After the students see that they get extreme results by increasing the perimeter while leaving the number of tiles the same, they may even propose laying out the tiles in a rectangular checkerboard pattern with the red squares being the tiles and the black squares being the holes. What happens if you break the group of tiles apart and lay out more than one floor with the same tiles?

3. After the possibility of laying out several floors occurs to the students, they will see that the pattern requiring you, the teacher, to lay out the most money for grout is the one in which each separate tile is a whole floor. Then each tile must be surrounded by a full width of grout, and there will be no economies generated by the fact that adjacent tiles in a floor share the grout between them. At the other extreme, those who imagine that they are spending their own money at the hardware store will see that a shape as nearly square as possible will be optimal.

4. In doing the calculations, students may propose that the analysis of the grout area can be simplified by thinking of each tile as if it were already completely surrounded by a strip of grout 1/8-inch wide. Then when two tiles are brought together side-by-side, the grout between them meets in a strip exactly 1/4 inches wide. The grout obtained in this way has to be adjusted by the extra amount needed around the edge of the floor. The extra amount at the edge of the floor is what brings the perimeter into play.

5. A useful tool for computation would be to use the letter s to denote the length of the side of the ceramic tile itself, and w to denote either the width of the grout or half the width of the grout, whichever the class settles on.

6. In calculating the extra grout required to make up the edge, special attention must be given to corners. By thinking of a room with an alcove on one side, the students will see that there are two types of corners, one type requiring a little extra grout and the other, the reentrant corners, requiring a little less. They may see that the corners requiring a little more grout always number four more than the corners requiring a little less—at least this is so if the floor has no holes in it.

7. Eventually, students could see that a 6 x 8 arrangement uses the least grout. They may eventually see that a square arrangement would be even better but can be accomplished only if the number of tiles is a perfect square. Now the nearest perfect square to 48 is 49. Students might examine the relationship between the 6 x 8 arrangement of 48 tiles and their arrangement into a 7 x 7 square with one corner tile missing. This could lead to an examination of relationships such as

$$(n - 1)(n + 1) = n^2 - 1.$$

If this point is reached, it would be useful to ask for the shape requiring the least grout for 52 tiles. That could lead to the general description of the optimal shape for any number of tiles.

Extensions:

1. By allowing holes in the floors, you get perimeters that are comprised of disconnected pieces. This has two effects. First, it increases the perimeter. Second, the number of normal angles less the number of reentrant angles may no longer net out to 4 (see *The Mathematics* section that follows).
2. Use of different polygons. Perhaps the most obvious extension in this way is to use rectangles, triangles, or hexagons. Other tessellations of the plane could be discussed. In fact, the grout problem could lead to a module on tessellations of the plane. This is an interesting geometric problem in its own right, and these detailed computations involving the perimeter become much less interesting in contrast to questions about the tessellations themselves.
3. Allow for three-dimensional shapes—how much grout does it take to build a solid shape out of bricks?
4. See what the effect is on the amount of grout needed if the area remains the same but the side of the tiles is doubled? Would you then double the spacing between the tiles as well?

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The Mathematics:

Let s be the length of the edge of the ceramic tile; in our example $s = 4$. Let w be half the width of the grout between tiles. In our example $w = 1/8$. The side of a tile surrounded by half the desired width of grout is $s + 2w$ (see Figure 1), and the area of this tile is $(s + 2w)^2$. The area of an extra half-width strip along a single side of the grout-surrounded tile is $(s + 2w)w$.

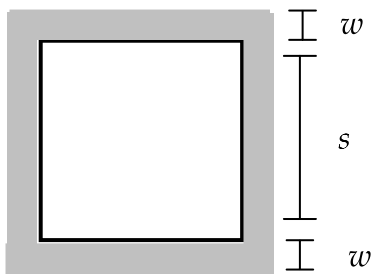

Figure 1: Width of tile

At a reentrant angle the extra grout needed along the two sides forming the angle is $(s + 2w)w + (s + 2w)w - w^2$, while the extra grout needed along the two sides forming a regular angle is $(s + 2w)w + (s + 2w)w + w^2$ (see Figure 2).

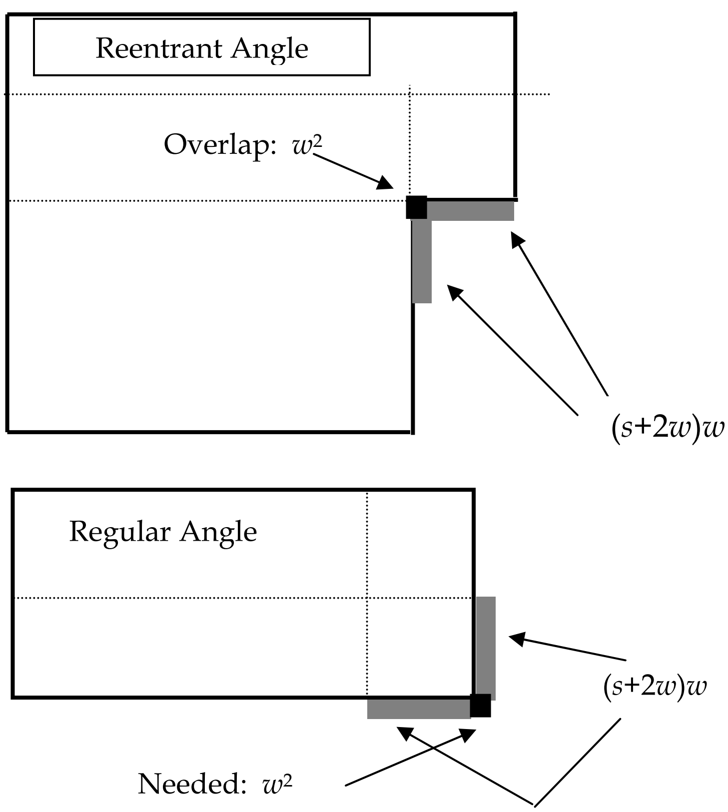

Figure 2: Grout at corners

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Why is the excess of normal angles over reentrant angles equal to 4? Imagine you start walking around the floor at its edge with the edge on your right. At a normal angle you will turn left, and at a reentrant angle you will turn right. If there were no reentrant angles, the floor would be rectangular and you would make four left turns to get back to your starting place. If there are any reentrant angles, each one of them, a right turn, would have to eventually be balanced by an extra left turn, so that you would still need a net of four left turns to get back to your start.

Thus, if the number of normal angles less the number of reentrant angles really is 4, then the extra grout needed around the perimeter of the floor is $p(s + 2w)w + 4w^2$, where p is the count of the number of tile sides along the entire perimeter.

The optimal shape, if the number of tiles is a perfect square, is a square. If the number of tiles is not a perfect square, it will be a number between two perfect squares. Starting with a square floor using the smaller perfect square, you can add tiles starting at a corner and continuing in a strip along the edge of the square until you use the prescribed number of tiles. What you end up with will be a floor using the least amount of grout.

Relationships to the Ohio Academic Content Standards, 2002:

Grades 5-7:

Number, Number Sense and Operations Standard
The student will be able to…

- Apply and explain the use of prime factorizations, common factors, and common multiples in problem situations.

Measurement Standard
The student will be able to…

- Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.

Patterns, Functions and Algebra Standard
The student will be able to…

- Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.
- Use symbolic algebra to represent and explain mathematical relationships

Mathematical Processes Standard
The student will be able to…

- Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.

Grades 8-10:

Measurement Standard
The student will be able to…

- Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.

Patterns, Functions and Algebra Standard
The student will be able to…

- Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

Mathematical Processes Standard
The student will be able to…

- Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- Apply mathematical knowledge and skills routinely in other content areas and practical situations.

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Relationships to the NCTM Principles and Standards, 2000:

Grades 6-8:

Number and Operations Standard
Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Algebra Standard
Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Understand patterns, relations, and functions.
- Use mathematical models to represent and understand quantitative relationships.

Geometry Standard
Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement Standard
Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving Standard
Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Connections Standard
Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Recognize and apply mathematics in contexts outside of mathematics.