

How Wide Is the River?

One-Page Overview

By Robert B. Brown, The Ohio State University

Topics:

Geometry, Indirect measurement

Levels:

Grades 5 – 8

Problem:

The purpose of this activity is to measure the distance between two points using indirect measurement without tools and where the student can only get to one of the two points.

Getting Started:

Pose the problem of determining the distance between two points, only one of which is accessible. See what the students propose to measure, but have in the back of your mind some actual possibilities, such as the width of a river or the height of a tall tree or building. After you have decided what to measure, break the students into groups and have them discuss their proposed method before going to the measurement site. Then go to the site for a reconnaissance. Let the students describe precisely the measurement that is desired, so that everyone is working toward the same thing.

Ohio Academic Content Standards, 2002

5-7		8-10		11-12	
1. Number, Number Sense and Operations	X	1. Number, Number Sense and Operations	x	1. Number, Number Sense and Operations	
2. Measurement	X	2. Measurement	X	2. Measurement	
3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	
4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra	
5. Data Analysis and Probability		5. Data Analysis and Probability		5. Data Analysis and Probability	
Mathematical Processes Problem Solving Connections Communication		Mathematical Processes Problem Solving Connections Communication		Mathematical Processes	

NCTM Principles and Standards, 2000

6-8		9-12	
1. Number and Operations	X	1. Number and Operations	
2. Algebra		2. Algebra	
3. Geometry	X	3. Geometry	
4. Measurement	X	4. Measurement	
5. Data Analysis and Probability		5. Data Analysis and Probability	
6. Problem Solving	X	6. Problem Solving	
7. Reasoning and Proof		7. Reasoning and Proof	
8. Communication	X	8. Communication	
9. Connections	X	9. Connections	
10. Representation		10. Representation	

Note: Capital X denotes major emphasis; lower case x denotes minor emphasis.

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<u>Topics:</u> Geometry, Indirect measurement	
<u>Levels:</u> Grades 5 – 8	<u>Timing:</u> Three hours. Allow an hour for a discussion of what could be measured indirectly, deciding what to measure, and how to measure it. Allow at least another hour for reconnaissance and taking the measurement. Finally, allow an hour for a discussion.
<u>Materials:</u> None	<u>Prerequisites:</u> Some intuitive geometric sense.

Problem:

The purpose of this activity is the indirect measurement (without tools) of the distance between two points where the student can get to only one of the points. An example is the width of a river where the student has to remain on one side. As time permits, you can later measure the same distance using more and more sophisticated measurement tools.

Goals:

- Understand that indirect measurement is often the only way to measure
- Devise techniques for indirect measurement
- See the need for standardized units of measurement
- Devise a method for reporting what are known to be inaccurate measurements
- Appreciate the fact that there is no way to measure anything with absolute precision

Big Ideas:

- Indirect measurement
- Units
- Geometric sense/intuition
- Precision

Procedure:

1. Pose the problem of determining the distance between two points, only one of which is accessible. See what the students propose as distances to measure, but have in the back of your mind some actual possibilities. The students are likely to suggest measuring the width of a lake or river, or the height of a mountain, building, or tree. Some of them may see that measuring the distance from the Earth to the sun or moon and measuring the altitude of an airplane above the ground are variations of this problem.
2. After you have decided what to measure (be sure to pick something that is not dangerous), put the students into groups and have them discuss their proposed method before going to the measurement site. Do not have the groups discuss their methods in front of the whole class at this point, because you want to encourage diverse methods.. Here are some methods that have been proposed for measuring the width of a river:
 - Base the measurement on congruent or similar triangles.
 - Use the geometry of 45-degree right triangles.
 - Locate a bridge across the river and count how many supports of some type are on the bridge.
 - Measure the distance between two supports on your side of the river and multiply.
 - Throw a rock on dry land, and step off the distance thrown. Then throw it across the river and estimate what portion of the width it covers. Multiply.
3. Go to the site for reconnaissance and measurement. This will probably force adjustments to proposed methods because of the peculiarities of the actual site. Let the students precisely describe the measurement so that everyone is working toward the same thing. For example, if you decide to measure the distance across a river, you need to agree on two points, one on each side, and focus on measuring the distance between those two points.

Tell the students that they are to find the measurement as closely as they can without using any tools whatsoever. They can move around one of the points but not the other and sight on each other, but they are not allowed to use any tape measures, string, graph paper, or other aids to measurement. Get the students to help set and agree upon the ground rules.

4. Reconvene for a classroom discussion of the measurements. Let each group report. There will be diversity of methods. Encourage a critical classroom discussion of each method.

Because no tools are allowed, the groups will make up their own unit of measurement. If the measurement is a distance, each group may pick one of its members to do pacing. Then they will report their measurement as so many of Julie's or Karl's paces. In order to compare the measurements of different groups they will have to decide how to change each measurement into an agreed upon unit. For example, they might decide to use the floor tiles in a hallway as a standard. It is important to see that they can compare the different measurements with each other by using tiles or another choice of a standard without knowing what the actual length of the standard is in terms of centimeters or inches.

Closure:

It is important to have a constructive classroom discussion of the methods and results of each group. Critically thinking about the diverse reasoning of all the groups will sharpen the geometric intuition of each student. Things to discuss:

- What are the geometric ideas underlying the various measurements that you made?
- Is ratio and proportion useful in indirect measurement?
- Have you ever been in a situation where you could not personally measure a distance that you wanted to know?
- What tools would you like to have to re-measure the distance?

The question of how to report an inaccurate measurement should also come up. Everyone will recognize that the measurements of the different groups will differ quite a bit, even after being reduced to some agreed-upon common unit. One way of addressing this problem is to round a result before reporting it. For example, instead of asserting that a distance is 453.7 meters, a group might report a measurement of 450 meters, or perhaps 450 meters give or take 20 meters.

Ask students to analyze the validity of the different methods used for finding the distance. Ask them how they might reduce the errors involved in each. Have students who understand explain to students who don't. If several understand but others don't, it is a good time to have a whole class discussion of the methods using drawings and other visual aids.

Many methods will lead to good estimates of the width. There is not just one *correct* method.

Which is the better problem: "find the width of the river" or "estimate the width of the river as accurately as you can?"

If you average together the estimates of all of the groups, would you be likely to get a more or a less accurate result? If "it depends," what does it depend on?

How do you explain wide discrepancies between the estimates of different groups of students?

Extensions:

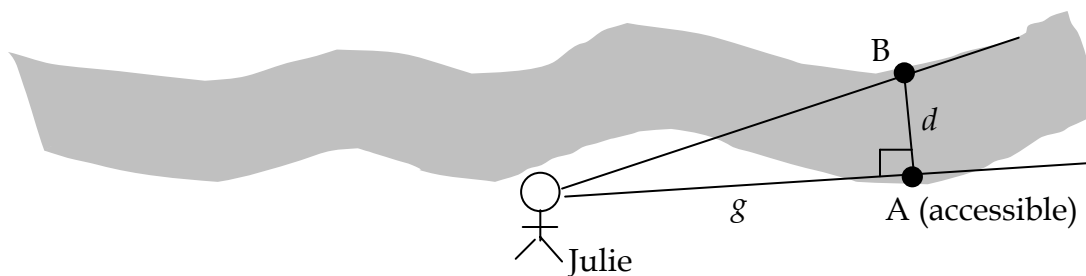
- On subsequent visits you can allow students to take the same measurement using increasingly sophisticated tools.
- On the first return visit to the site you could give each group a piece of string, say roughly 2 meters long, and a straight rod to sight along.
- On the next visit you could allow, in addition, a meter stick and graph paper, or maybe a protractor, trundle wheel, or long tape measure. Give them a chance to agree on what tools should be allowed.
- You could wrap up the activity with a last visit accompanied by a surveyor with modern equipment. Where do you find a surveyor for free? How about a local university, utility company, or highway department? Modern laser-ranging surveying equipment can measure a distance of several hundred meters to an accuracy of one millimeter or less and can measure angles with comparable accuracy. Then you have a very accurate measure, and each group can see how close they were able to come.

The Mathematics:

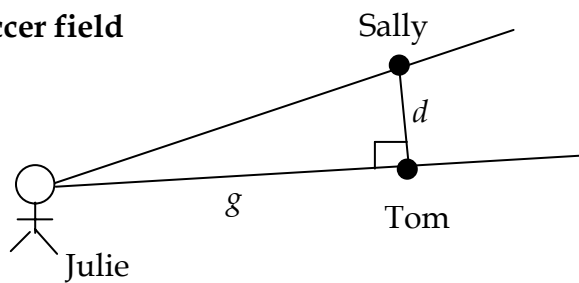
Here are three of the many ways to estimate the width of the river and then, three ways to estimate the width of a lake:

River Technique #1. This is a method using the geometry of **congruent triangles** but no tools. Do your best to pace off by sight a line perpendicular to the distance d that is to be measured. Julie (see figure) then starts at the accessible point and holds one hand out in front of her face, backing up until her hand just spans the distance to be measured. Then Karl paces off the distance g from Julie back to the accessible point. Next the group goes to a nearby soccer field. Tom stands on a spot. Then Karl paces off distance g away from Tom. Julie stands where Karl finishes pacing and holds up her hand in front of her face again. Then Sally paces away from Tom at a right angle to the line from Tom to Julie until Julie's hand just spans the distance from Tom to Sally. Sally then knows how many paces she took. That is, the number of Sally's paces is the width across the river, because the two triangles illustrated here are congruent.

Across the river

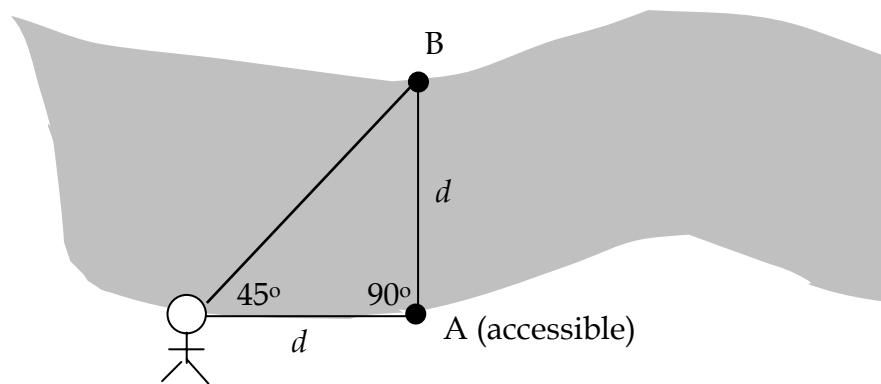


On the soccer field



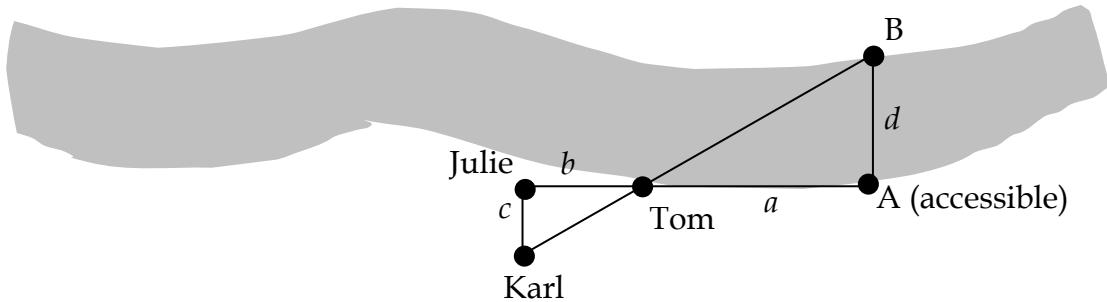
River Technique #1: Using congruent triangles

River Technique #2. Another technique involves trying to lay out a **square** with one side being the distance to be measured, d , and another side along the accessible shore. The square is laid out by a person walking from point A along the shore at right angles to the width of the river until the angle over to the inaccessible point, B, is estimated to be 45 degrees. The two sides of the square have the same length and, since you know how many paces is the side along the shore, you know that the distance across the river is the same length.



River Technique #2: Laying out a square

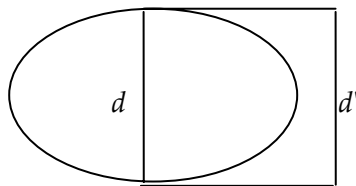
River Technique #3: This method uses **similar triangles**. Pace along a convenient line along the accessible side of the river. Stop and let Julie stand where you stopped. Then pace away from the river on a line perpendicular to the line you just walked. Stop again at a convenient point and let Karl stand where you stopped. Then have Karl sight steadily to the inaccessible point on the other side of the river and have Tom walk from the accessible point toward Julie until he is in Karl's line of sight. Now you have set up two similar triangles. Have Sally now pace off the three distances a , (A to Tom), b (Tom to Julie), and c (Julie to Karl). Because all of the ratios between two corresponding sides in similar triangles are equal, you can solve for the unknown width.



$$\frac{d}{a} = \frac{c}{b}$$

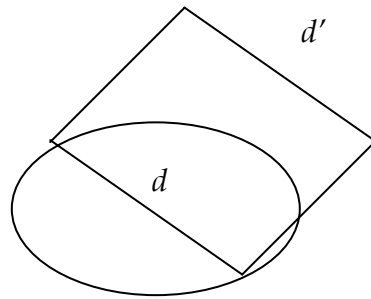
River Technique #3: Using similar triangles

Lake Technique #1: Here is a common method of measuring the **distance across a lake** without using tools. Do your best to lay off, by sight, perpendiculars to the actual distance d . Then pace off a representative distance d' . If this were done perfectly, the straight-line figure would be a rectangle, and the distances d and d' , which are along opposite sides, would be the same.



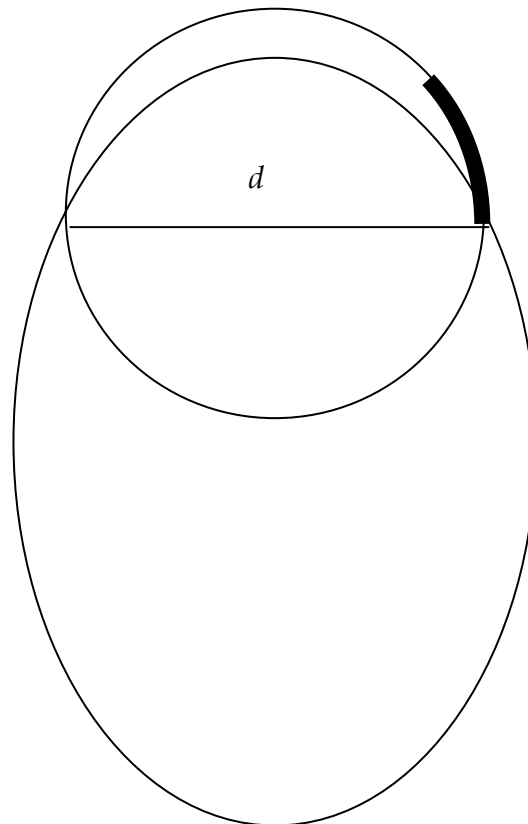
Lake Technique #1: Using a rectangle

Lake Technique #2: However, the shore of the lake may be irregular, parts may be inaccessible, or the terrain may make pacing d' difficult. Here is a slightly modified method that uses a **parallelogram** instead of a rectangle. Again, d and d' will be equal, because they measure opposite sides of a parallelogram.



Lake Technique #2: Using a parallelogram

Lake Technique #3: A more complicated technique that illustrates the range of possibilities is the following. Imagine a **circle** whose diameter stretches between the two points that you are measuring. Then you pace off the distance along an arc of the circle, estimate what proportion of the whole circle is your arc, and finally use the relation between the length of the circumference and the length of the diameter to estimate the length of the diameter. There is plenty of room for error, because this is a tall order to carry out by sight alone. In the diagram below, the heavy arc was paced off at 40 paces. The arc was estimated to be $1/8$ of the circle, giving $40 \times 8 = 320$ paces for the circle. Using the approximation that the circumference is about 3 times as long as the diameter, the distance between the two points was estimated at $320/3 = 107$ paces.



Lake Technique #3: Using a circle

Relationships to the Ohio Academic Content Standards, 2002:

Grades 5-7:

Number, Number Sense and Operations Standard

The student will be able to...

- Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents.

Measurement Standard

The student will be able to...

- Select appropriate units to measure angles, circumference, surface area, mass and volume, using:
 - U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;
 - metric units; e.g., square meters, kilograms and other units as appropriate.
- Select a tool and measure accurately to a specified level of precision.
- Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.

Geometry and Spatial Sense Standard

The student will be able to...

- Describe and use the concepts of congruence, similarity and symmetry to solve problems.
- Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing.

Mathematical Processes Standard

The student will be able to...

- Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
- Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
- Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.

Grades 8-10:

Number, Number Sense and Operations Standard

The student will be able to...

- Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions

Measurement Standard

The student will be able to...

- Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.
- Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.
- Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.

Geometry and Spatial Sense Standard

The student will be able to...

- Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.

Mathematical Processes Standard

The student will be able to...

- Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- Apply mathematical knowledge and skills routinely in other content areas and practical situations.
- Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Relationships to the NCTM Principles and Standards, 2000:

Grades 6-8:

Number and Operations Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Compute fluently and make reasonable estimates.

Geometry Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Apply transformations and use symmetry to analyze mathematical situations.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problem.

Communication Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Organize and consolidate their mathematical thinking through communication.
- Analyze and evaluate the mathematical thinking and strategies of others.

Connections Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Recognize and use connections among mathematical ideas.
- Recognize and apply mathematics in contexts outside of mathematics.