

Magic Birthday Cards

One-Page Overview

By Sigrid Wagner, The Ohio State University

Topics:

Patterns, Problem solving, Discovery, Inquiry

Levels:

Grades 5 – 8

Problem:

Using the Magic Birthday cards, guess your classmates' birthdays.

Getting Started:

Have students take turns selecting the cards on which their birthday appears. First, have them select the cards on which the number of their birth month appears; next, the cards on which the day of the month appears. After each selection, display the cards in random order and discuss the various possibilities before "guessing" the correct answer. As individual students discover a strategy for determining the correct numbers, let them guess a classmate's birthday.

Ohio Academic Content Standards, 2002

5-7		8-10		11-12	
1. Number, Number Sense and Operations	X	1. Number, Number Sense and Operations	x	1. Number, Number Sense and Operations	
2. Measurement		2. Measurement		2. Measurement	
3. Geometry and Spatial Sense		3. Geometry and Spatial Sense		3. Geometry and Spatial Sense	
4. Patterns, Functions and Algebra	X	4. Patterns, Functions and Algebra	x	4. Patterns, Functions and Algebra	
5. Data Analysis and Probability		5. Data Analysis and Probability		5. Data Analysis and Probability	
Mathematical Processes Problem Solving Reasoning		Mathematical Processes Problem Solving Reasoning		Mathematical Processes	

NCTM Principles and Standards, 2000

6-8		9-12	
1. Number and Operations	X	1. Number and Operations	
2. Algebra	X	2. Algebra	
3. Geometry		3. Geometry	
4. Measurement		4. Measurement	
5. Data Analysis and Probability		5. Data Analysis and Probability	
6. Problem Solving	X	6. Problem Solving	
7. Reasoning and Proof	x	7. Reasoning and Proof	
8. Communication		8. Communication	
9. Connections		9. Connections	
10. Representation		10. Representation	

Note: Capital X denotes major emphasis; lower case x denotes minor emphasis.

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<u>Topics:</u> Patterns, Problem solving, Discovery, Inquiry	
<u>Levels:</u> Grades 5 – 8	<u>Timing:</u> Discovering the trick can be done in one class period or extended to short sessions over several days. Explanations and extensions will take more time and depend on the level of the students.
<u>Materials:</u> Magic Birthday Cards (one display set, plus a set for each student)	<u>Prerequisites:</u> Simple whole number addition Place value Binary numeration

Problem:

Using the Magic Birthday cards, guess your classmates' birthdays.

Goals:

- Have some fun “guessing” birthdays (month and day)
- Discover how birthday cards work
- Figure out why the birthday cards work
- Extend the ideas to make other sets of cards

Big Ideas:

- Place value
- Numeration in other bases
- Representation
- Number patterns

Procedure:

1. Before you begin the activity, caution students not to blurt out the "trick" — explain that they will be able to show that they know the trick without giving the secret away to their classmates.
2. Have students take turns selecting the cards on which their birthday appears. First, have them select the cards on which the number of their birth month appears; next, the cards on which the day of the month appears. After each selection, display the cards in random order and discuss the various possibilities before "guessing" the correct answer. As individual students discover a strategy for determining the correct numbers, let them guess a classmate's birthday. You can use a few theatrics (photographic memory, mental telepathy) to dramatize the magical aspect of the cards.
3. Encourage questioning, conjecturing, and looking for patterns. Students will eventually notice an "order" to the cards and should observe patterns in the numbers that occur on each card. Have them work in small groups to figure out how, and then why, the cards work. They should eventually see that the magic number is the first number that appears on all of the selected cards and that the easiest way to determine this number is to add the upper left-hand corner numbers of the selected cards.
4. Lead students as far as they can go in understanding why the cards work. If they are familiar with numerals in bases other than 10, discuss binary numerals (base 2). Numbers appear on the k card if there is a 1 in the k^{th} place of their binary expansion.

Example:

$$22_{\text{ten}} = \begin{matrix} 1 & 0 & 1 & 1 & 0 \\ (16\text{s}) & (8\text{s}) & (4\text{s}) & (2\text{s}) & (1\text{s}) \end{matrix} \text{ two}$$

So 22 appears on the 16, 4, and 2 cards but not on the 8 and 1 cards.

Emphasize that mathematically the cards that do not contain the number are just as important as the cards that do. Have students write the binary expansions of the numbers 1–31 and verify that each number appears on the appropriate cards and not on the other cards. You can relate 1 and 0 to on/off switches in computers.

Evaluation:

How quickly do students figure out a strategy? How well are they able to explain their reasoning? Can they make their own set of magic birthday cards? Can they think of ways to extend their ideas? Can they make an expanded set of cards? Can they make a set of cards in another base?

Extensions:

- Can we extend the range of numbers beyond 31? Yes, but now we need a 32 card and bigger 1, 2, 4, 8, and 16 cards. Have students make the new cards.
- Can we use a base other than 2 to construct magic cards? Yes, but how does this change the problem?

Set of 3 cards: Pick a number from 1 to 7

1	3
5	7

2	3
6	7

4	5
6	7

Set of 4 cards: Pick a number from 1 to 15

1	3	5
7	9	11
13	15	

2	3	6
7	10	11
14	15	

4	5	6
7	12	13
14	15	

8	9	10
11	12	13
14	15	

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4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

Relationships to the Ohio Academic Content Standards, 2002:

Grades 5-7:

Number, Number Sense and Operations Standard

The student will be able to...

- Apply number system properties when performing computations.

Patterns, Functions and Algebra Standard

The student will be able to...

- Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.

Mathematical Processes Standard

The student will be able to...

- Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
- Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
- Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.

Grades 8-10:

Mathematical Processes Standard

The student will be able to...

- Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

Relationships to the NCTM Principles and Standards, 2000:

Grades 6-8:

Number and Operations Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Compute fluently and make reasonable estimates.

Algebra Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Understand patterns, relations, and functions.

Problem Solving Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Apply and adapt a variety of appropriate strategies to solve problems.

Reasoning and Proof Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Make and investigate mathematical conjectures.