Ohio's Systemic Initiative Discovery

Pythagorean Theorem

One-Page Overview

By Robert B. Brown, The Ohio State University

Topic:

Geometry

Levels:

Grades 5 – 8

Problem:

This activity shows how cutting geometric shapes and rearranging the pieces leads to some standard area formulas and to the Pythagorean Theorem.

Getting Started:

Ask the students what they think about the assertion that, if a region is cut up into pieces that don't overlap, the area of the original region is the sum of the areas of all of its pieces. They will probably agree. Ask them to draw a rectangle and tell you what its area is. Ask them to cut it in half along a diagonal. What can they conclude about the area of one of the resulting triangles? Does this coincide with their understanding of what the area of a triangle is?

Ohio Academic Content Standards, 2002

5-7		8-10		11-12	
1. Number, Number Sense and Operations		1. Number, Number Sense and Operations		1. Number, Number Sense and Operations	
2. Measurement	x	2. Measurement	x	2. Measurement	
3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	
4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra	
5. Data Analysis and Probability		5. Data Analysis and Probability		5. Data Analysis and Probability	
Mathematical Processes Problem Solving		**Mathematical Processes** Problem Solving		**Mathematical Processes**	

NCTM Principles and Standards, 2000

6-8		9-12	
1. Number and Operations		1. Number and Operations	
2. Algebra		2. Algebra	
3. Geometry	X	3. Geometry	X
4. Measurement		4. Measurement	
5. Data Analysis and Probability		5. Data Analysis and Probability	
6. Problem Solving	X	6. Problem Solving	X
7. Reasoning and Proof		7. Reasoning and Proof	
8. Communication		8. Communication	
9. Connections		9. Connections	
10. Representation		10. Representation	

<u>Note</u>: Capital X denotes major emphasis; lower case x denotes minor emphasis.

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Pythagorean Theorem

By Robert B. Brown, The Ohio State University

Topics: Geometry	
Levels: Grades 5 - 8	*Timing:* One hour
Materials: Paper Rulers, preferably one per student or one per group Scissors, preferably one per student or one per group	*Prerequisites:* Knowledge of areas of rectangles and triangles

Problem:

This activity shows how cutting geometric shapes and rearranging the pieces leads to some standard area formulas and to the Pythagorean Theorem.

Goals:

- Increase familiarity with areas of triangles and rectangles
- Show that looking at something in two different ways yields results
- Practice using areas
- Establish the all-important Pythagorean Theorem

Big Ideas

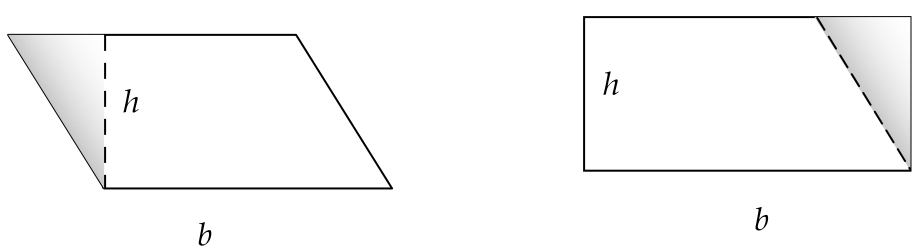
- Areas of triangles
- Areas of rectangles
- Two different ways yields results
- Using areas
- Pythagorean Theorem

Procedure:

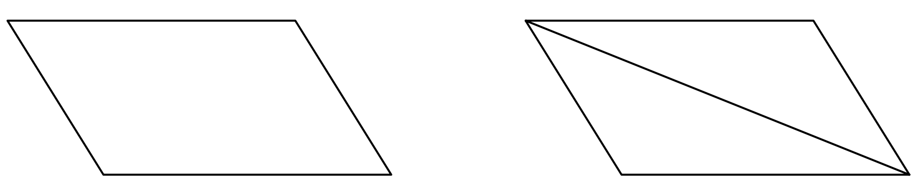
1. Ask the students what they think about the assertion that if a region is cut up into pieces that don't overlap, then the area of the original region is the sum of the areas of all of its pieces. They will probably agree. Ask them to draw a rectangle and tell you what its area is. Ask them to cut it in half along a diagonal. What can they conclude about the area of one of the resulting triangles? Does this coincide with their understanding of the area of a triangle?

The area of the rectangle is *bh*. The triangles are equal in area, so one of them has area *1/2(bh).*

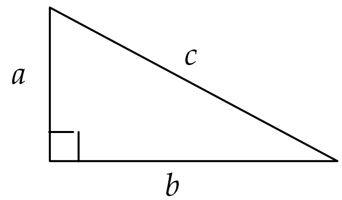
2. Have them see that the area of a parallelogram is also *bh*, the same as a rectangle, by having them cut off one slant end of a parallelogram and move it to the other end so as to form a rectangle.



3. Once parallelograms are under control, a parallelogram can be cut along a diagonal to see that for all triangles the area is *1/2(bh).*



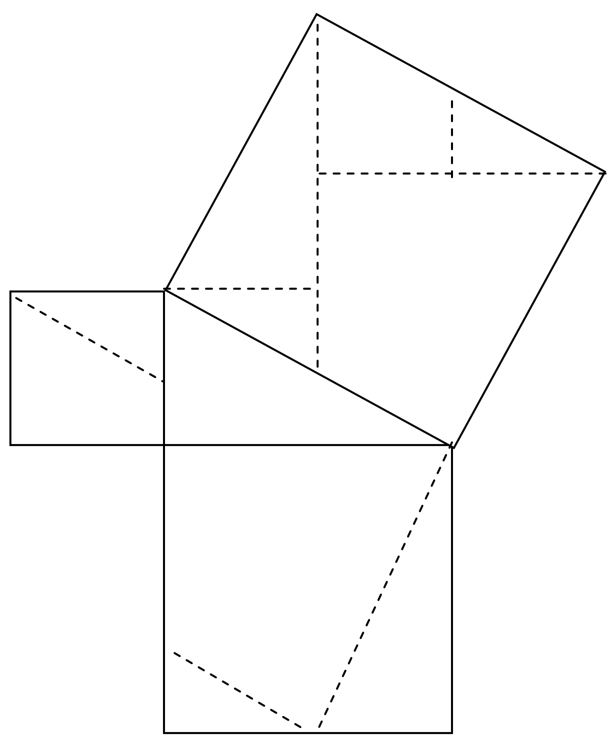
4. Now you are ready to have the students deal with the Pythagorean Theorem.



Here is a *right* triangle. Sides of lengths *a* and *b* come together at the right angle, and the third side, of length *c*, is the hypotenuse. The Pythagorean Theorem guarantees that the areas of squares of sides *a* and *b* add up to the area of a square with side *c*. Algebraically this means that $a^2 + b^2 = c^2$.

Below are two ways of seeing it.

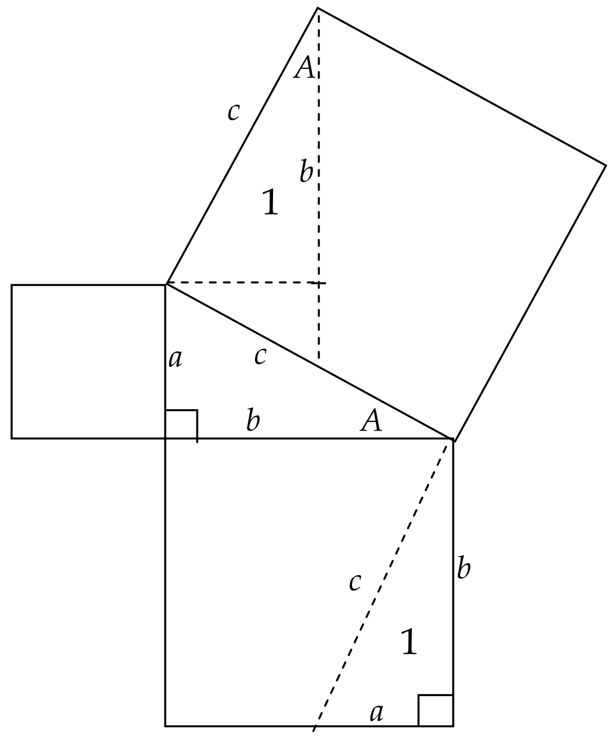
5. Here is one way to "see" the Pythagorean Theorem. In this figure the pieces that make up the large square with area c^2 can be rearranged to form the two small squares.



Here is one proof:

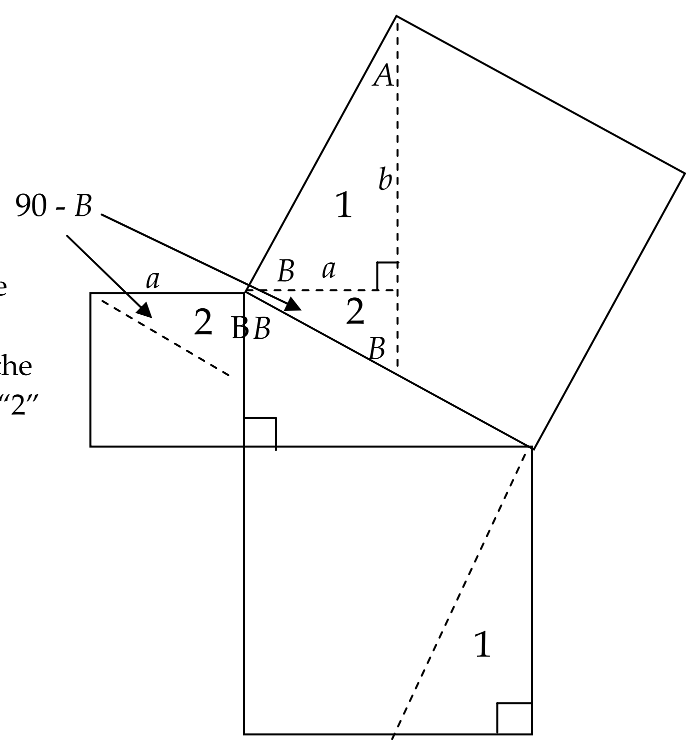
Draw a right triangle with squares on each side. On the square with side *b*, mark off a segment, *a*, as shown. Because of the SAS property, the hypotenuse of this new triangle must be *c*, since the triangle is congruent to the original.

In the square with side *c*, construct a segment that makes an angle *A*, as shown. Mark off a length *b* on this segment. Because of SAS, the triangle formed is, again, congruent to the original triangle, and therefore the new triangles are congruent to each other (1).



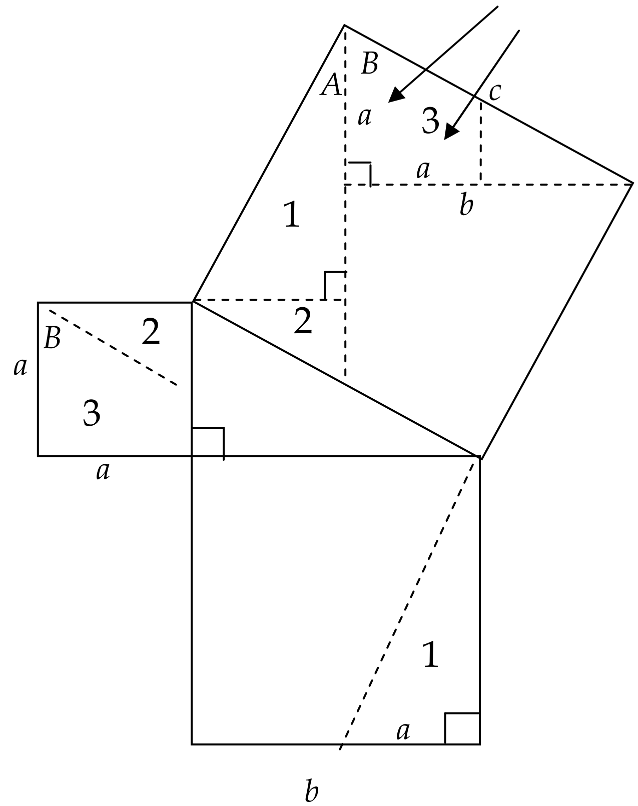
Next, notice angle *B* of triangle 1 at the top. The complement of angle *B* is 90-*B* in triangle 2.

Construct a segment in the "*a*" square that makes an angle measuring 90-*B*. Since this angle is in a right triangle, the other angle measures *B*. By ASA, the "2" triangles are congruent.

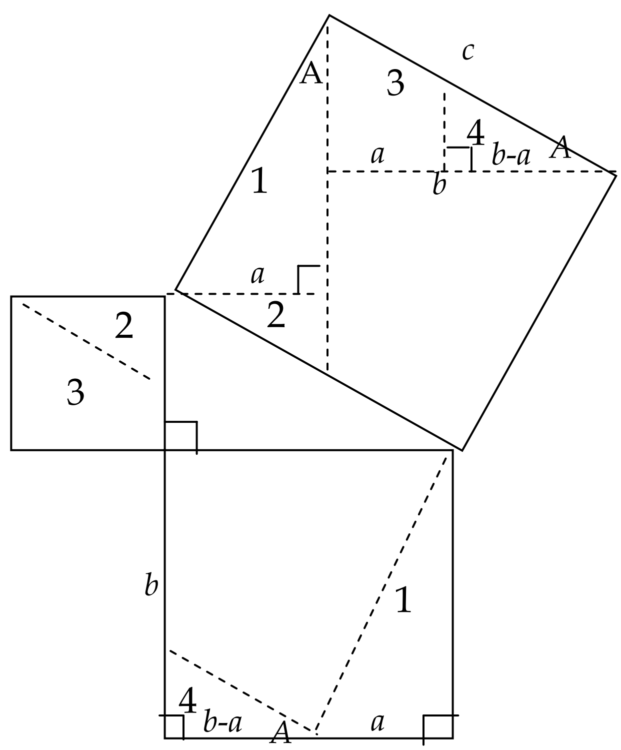


Mark off a length *a* on triangle 1 in the “c” square as shown. Construct a segment perpendicular to this segment. This new triangle has sides *a* and *c* with included angle *B*. By SAS, this triangle is congruent to the original triangle.

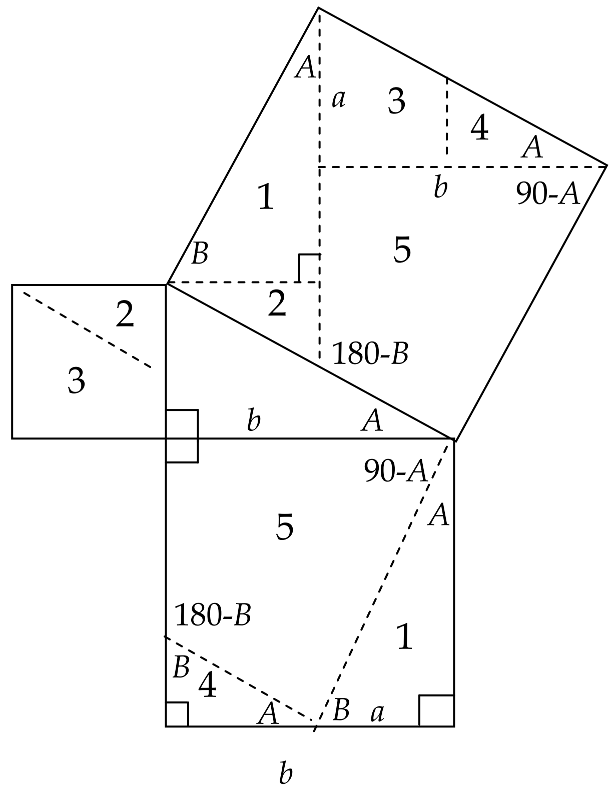
Construct a segment *a* on segment *b* of this triangle and draw a perpendicular from *b* to *c*. Looking at this quadrilateral (3) and the one in the “*a*” square, we see that they are congruent since corresponding angles are congruent (therefore the figures are proportional) and at least one pair of sides is congruent.



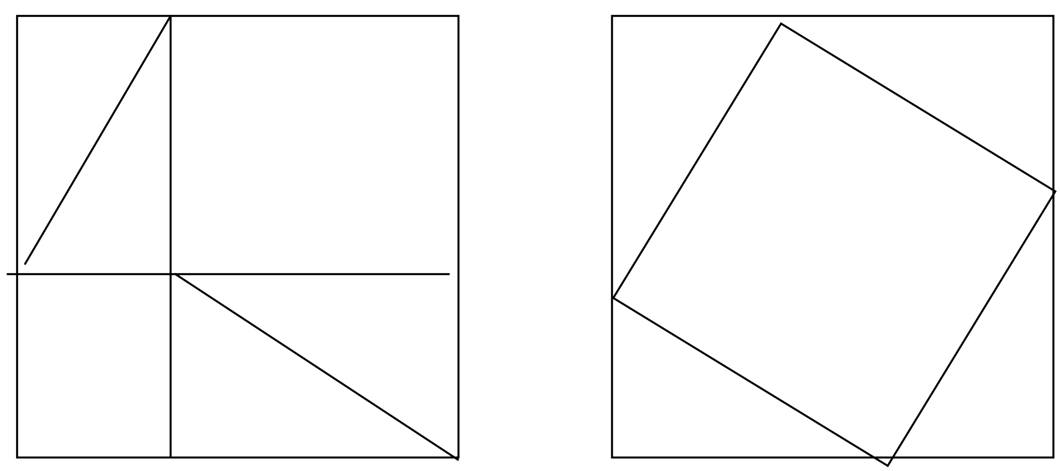
Construct a segment in square “*b*” such that the new triangle has an angle *A* as shown. By ASA, this triangle is congruent to the triangle (4) in square “*c*.”



Finally, the quadrilaterals marked 5 are congruent since corresponding angles are congruent and at least one corresponding side is congruent.



6. Here is a second way. Both of these figures show a square of side $a + b$ with four identical triangles inside. The triangles are arranged differently in the two big squares. Nevertheless, if the areas of the four triangles are subtracted from the areas of the big squares, the remaining areas are the same. In the left square, the remaining area is $a^2 + b^2$ while in the right square the remaining area is c^2. Therefore, $a^2 + b^2 = c^2$.



Relationships to the Ohio Academic Content Standards, 2002:

Grades 5-7:

Measurement Standard

The student will be able to…

- Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.

Geometry and Spatial Sense Standard

The student will be able to…

- Describe and use the concepts of congruence, similarity and symmetry to solve problems.

Mathematical Processes Standard

The student will be able to…

- Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.

Grades 8-10:

Measurement Standard

The student will be able to…

- Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.

Geometry and Spatial Sense Standard

The student will be able to…

- Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.

Mathematical Processes Standard

The student will be able to…

- Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose a method for obtaining this information, and set limits for acceptable solution.

Relationships to the NCTM Principles and Standards, 2000:

Grades 6-8 and 9-12 :

Geometry Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Problem Solving Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to…

- Build new mathematical knowledge through problem solving.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.