

Similar Triangles

One-Page Overview

By Robert B. Brown, The Ohio State University

Topics:

Geometry

Levels:

Grades 7 – 11

Problem:

What would be a reasonable definition of similarity? What are sufficient conditions that two triangles be similar? Quadrilaterals? Other figures?

Getting Started:

Everyone should draw a triangle on a piece of paper. Measure the lengths of its sides, and multiply these lengths by the same non-whole number. Have each student construct from straws and string a triangle with sides of the new lengths. What do you see?

Ohio Academic Content Standards, 2002

5-7		8-10		11-12	
1. Number, Number Sense and Operations		1. Number, Number Sense and Operations		1. Number, Number Sense and Operations	
2. Measurement		2. Measurement		2. Measurement	
3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	X	3. Geometry and Spatial Sense	x
4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra		4. Patterns, Functions and Algebra	
5. Data Analysis and Probability		5. Data Analysis and Probability		5. Data Analysis and Probability	
Mathematical Processes Connections		Mathematical Processes Connections		Mathematical Processes Connections	

NCTM Principles and Standards, 2000

6-8		9-12	
1. Number and Operations		1. Number and Operations	
2. Algebra		2. Algebra	
3. Geometry	X	3. Geometry	X
4. Measurement		4. Measurement	
5. Data Analysis and Probability		5. Data Analysis and Probability	
6. Problem Solving		6. Problem Solving	
7. Reasoning and Proof		7. Reasoning and Proof	
8. Communication		8. Communication	
9. Connections	x	9. Connections	x
10. Representation		10. Representation	

Note: Capital X denotes major emphasis; lower case x denotes minor emphasis.

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<u>Topics:</u> Geometry	
<u>Levels:</u> Grades 7 – 11	<u>Timing:</u> One hour
<u>Materials:</u> Straws String Overhead projector, transparencies, and pens	<u>Prerequisites:</u> Familiarity with triangles and other polygons Angle measure Proportionality

Problem:

What would be a reasonable definition of similarity? What are sufficient conditions that two triangles be similar? Quadrilaterals? Other figures?

Goals:

- Investigate the foundations of similarity
- Practice using the concept of proportionality
- See that many polygons with sides of fixed lengths need not be rigid
- See what minimal conditions will guarantee similarity of polygons with few sides

Procedure:

1. Have each student draw a triangle on a piece of paper, measure the lengths of its sides, and multiply these lengths by the same non-whole number.
2. Have each student construct from straws and string a triangle with sides of the new lengths.
3. Have one student draw the original triangle on a transparency, and put the transparency and the straw triangle on the overhead simultaneously. Ask the students to talk about what they see.
4. Does it help to “see” anything by keeping the corresponding sides of the two triangles parallel?
5. Will this procedure lead to the same result with a quadrilateral? Hexagon? Other polygons? The students will notice that only the triangle holds its shape.
6. The technical term for having the same shape is “similar.”
7. Break the students into groups for 5 minutes and ask each group to come up with a definition for similar figures, one that would apply to all shapes, not just triangles.
8. Reconvene the whole class and let each group report. The general idea is that for two figures to be similar, corresponding angles should be congruent and lengths of corresponding parts should be proportional.
9. After the class has agreed on a working definition of similarity, try it out on some new figures such as circles, cubes, and other two-dimensional figures. The definition should work for figures of any dimension.
10. Are all circles similar? Are all line segments similar? Are all angles similar?

Extensions:

- What conditions on three pairs of corresponding parts of a pair of triangles would guarantee similarity?
- Could you guarantee similarity of a pair of triangles by conditions on just two pairs of corresponding parts?
- How could you handle quadrilaterals? Pentagons? Hexagons?
- In one class a courtroom drama was set up with a defendant, prosecutor, judge, and jury. The defendant was overheard claiming that two triangles would be similar if two pairs of corresponding angles were congruent. The prosecution produced an example with two pairs of corresponding angles that were congruent, but corresponding pairs of sides were not proportional. The triangles produced in

evidence were two triangles on a sphere, each with two right angles at the equator. An objection was made that comparisons cannot be made between plane and spherical geometry. The drama ended when a strong objection was made to its all-male cast!

Closure:

Similarity is one of the key concepts in geometry. It underlies the whole subject of trigonometry.

The Mathematics:

Seeing one student's original triangle and new straw triangle on the overhead at the same time will show everyone that these two triangles have the same shape. For triangles the fact that corresponding sides are proportional guarantees that corresponding angles are congruent and that the triangles have the same shape.

Proportionality of corresponding sides is not enough to guarantee that two quadrilaterals are similar because quadrilaterals are not rigid figures. For example, one could take a square, double the length of each side and then squeeze the square a bit so that it becomes a diamond. The diamond has sides proportional to the sides of the square but it has a different shape. The same flexibility of quadrilaterals is enjoyed by all polygons with more than three sides.

An intuitive definition of similarity is that two figures are similar if all pairs of corresponding angles are congruent and all lengths of pairs of corresponding parts are proportional. This notion of similarity should serve the students well through quite a bit of advanced mathematics. It works for figures of all dimensions. In particular, it shows that any two line segments are similar, because there are no angles to worry about, and only one pair of lengths to compare. And one length is always proportional to another length when there is no additional comparison of lengths to be checked against.

Any two circles are also similar, although it is not clear what are the lengths and angles to be compared. One could always resort to the proportionality of chords with congruent central angles in the two circles. One difficulty with a circle is to define what a circle is. The circle technically is just the circumference. Neither the center nor any other point inside the circle is part of the circle itself.

What conditions on three pairs of corresponding parts of two triangles would guarantee similarity?

SSS: Three pairs of corresponding sides proportional

AAA: Three pairs of corresponding angles congruent

SAS: Two pairs of corresponding sides proportional and the included angles congruent

ASA: Two pairs of corresponding angles congruent and the common sides proportional.

AAS: Two pairs of corresponding angles congruent and a side (not common) is proportional

ASS: (doesn't work)

Could you guarantee similarity of a pair of triangles by conditions on just two corresponding parts?

How could you handle quadrilaterals? Pentagons? Hexagons?

Quadrilaterals: Four pairs of corresponding sides proportional and one pair of corresponding angles congruent, sort of SSASS.

Would SSAS work for quadrilaterals?

Relationships to the Ohio Academic Content Standards, 2002:

Grades 5-7:

Geometry and Spatial Sense Standard

The student will be able to...

- Use proportions to express relationships among corresponding parts of similar figures.
- Describe and use the concepts of congruence, similarity and symmetry to solve problems.
- Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing.

Mathematical Processes Standard

The student will be able to...

- Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies

Grades 8-10:

Geometry and Spatial Sense Standard

The student will be able to...

- Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.

Relationships to the NCTM Principles and Standards, 2000:

Grades 6-8 and Grades 9-12:

Geometry Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Connections Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.