

Treasure Map Problem

It is not quite clear how it happened, but here you are on Treasure Island, holding an old, weatherbeaten, but still legible map. The map tells you to find a lone pine tree at the edge of the beach, and a large pinkish boulder nearby. You worry about whether they are still there, but you manage to find both the tree and the boulder, and you continue reading the map.

It says: "Call the tree P , and call the boulder B . Put yourself on the line PB , somewhere between the two, and walk out on the beach toward the water. Stop wherever you like, and mark the point W where you stopped. Now, carefully locate two points X and Y on the inland side of PB , as follows: $PX = PW$ and angle WPX is a right angle. $BY = BW$ and angle WBY is a right angle. Dig at the midpoint of XY and you'll find the treasure there." These directions are not so hard to follow, but wait a minute: just what is going on here? Isn't the bit about picking any point W just a little bit vague?

Well, you try the directions anyway, you find that midpoint, and what do you know, there is a small, elegant, wooden chest. Locked. You will have to wait until you have something to open it with, but in the meantime, since – for all you know – you will be stuck on this island for a while, you decide to figure out how these directions work.

You start over, and walk out to another point W , follow the directions, and amazingly, you wind up right back at the treasure chest. What is going on? How can this be? Since you can stop anywhere you want, where might be some clever places to put point W so you can get some ideas how this works?