

Treasure Map Problem, p. 5

Conjecture: The treasure is at the center of a square whose side is  $\overline{PB}$ . (The center is the midpoint of diagonal  $\overline{BD}$ .)

We prove that the midpoint of  $\overline{XY}$ , where the treasure is, according to the map's directions, is also the midpoint of  $\overline{BD}$ , thus proving our conjecture.

We draw:

- an arbitrary point  $W$ , and from it the segment  $\overline{XY}$ , according to the map's directions;
- the square  $PBCD$ ,
- diagonal  $\overline{BD}$  of the square, and
- $\overline{XD}$  and  $\overline{YC}$ .

We label the intersection of  $\overline{XY}$  and  $\overline{BD}$   $Z$ .

Proof: We show that  $X$  is the midpoint of  $\overline{XY}$  and thus is the location of the treasure. We then show that  $Z$  is also the midpoint of  $\overline{BD}$ , and thus that the treasure is at the center of the square.

- $\triangle PXD \cong \triangle PWB$ , by SAS:  $\overline{PX} \cong \overline{PW}$ ,  $m\angle XPD = m\angle WPB = 90 - m\angle XPB$ , and  $\overline{PD} \cong \overline{PB}$ .
- Similarly,  $\triangle CYB \cong \triangle PWB$ , by SAS:  $\overline{YB} \cong \overline{WB}$ ,  $m\angle CBY = m\angle PBW = 90 - m\angle YBP$ .
- $\therefore \triangle PXD \cong \triangle CYB$ , and we note that  $\angle PDX \cong \angle CBY$  and  $\overline{DX} \cong \overline{BY}$ , CPCTC.
- $\triangle XZD \cong \triangle YZB$ , by SAA:  $\overline{DX} \cong \overline{BY}$ ,  $m\angle XDZ = m\angle YBZ = 45 - m\angle PDX$  ( $= 45 - m\angle CBY$ ), and  $\angle XZD \cong \angle YZB$  (vertical angles).
- $\therefore \overline{XZ} = \overline{YZ}$ , CPCTC, and  $Z$  is the midpoint of  $\overline{XY}$ .
- $\therefore$  The treasure is at  $Z$ .
- But  $\overline{DZ} \cong \overline{BZ}$ , CPCTC, and thus  $Z$  is the midpoint of  $\overline{BD}$ .
- Thus the treasure is at the midpoint of  $\overline{BD}$ , the center of square  $PBCD$ , q. e. d.

This means that the location of the treasure depends only on  $\overline{PB}$ , and not on where you walk.

P.S. When you finally get the treasure chest open, what's in it?