Slides from Generating Polynomial Equations for Given Roots (Tutorial 13)

Generating Polynomial Equations for Given Roots

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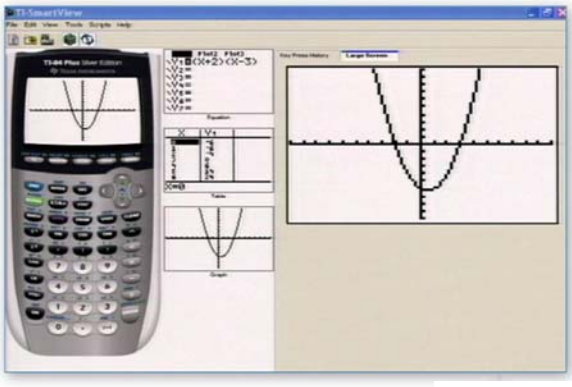
This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

A Function in Factored Form

$f(x) = (x + 2)(x - 3)$

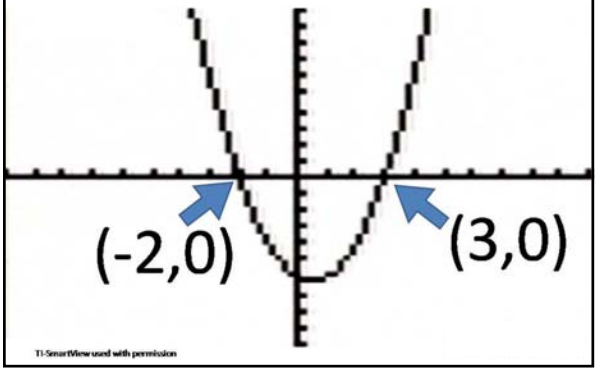
Graph the function to find the roots

A *root* is an x-value that makes a function equal to zero. A root can also be called a *zero* of the function



TI-SmartView used with permission

$y = (x + 2)(x - 3)$



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The function

$f(x) = (x + 2)(x - 3)$

has

roots: $x = -2$ and $x = 3$

factors: $(x+2)$ and $(x-3)$

This is a reliable pattern! Whenever we find a root for a function, there should be a factor that is *x* minus that root's value.

Factored Form ➡ *General Quadratic Form*

Multiply the factors together and simplify

$f(x) = (x + 2)(x - 3)$

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$f(x) = (x+2)(x-3)$

Multiply using the distributive property.

$(x+2)(x-3)$

$f(x) = (x+2)(x-3)$

Multiply using the distributive property.

$(x+2)(x-3)$

$x^2 - 3x + 2x - 6$

$f(x) = (x+2)(x-3)$

Multiply using the distributive property.

$(x+2)(x-3)$

$x^2 - 3x + 2x - 6$

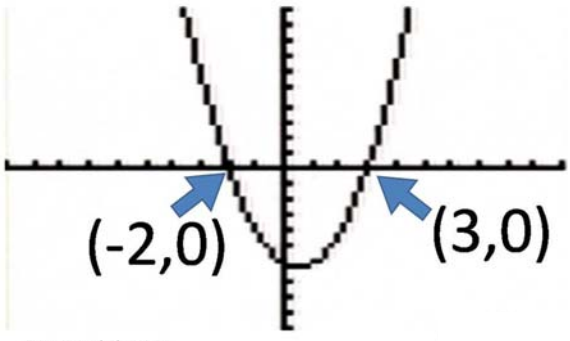
Combine Like Terms

$x^2 - 3x + 2x - 6$

$x^2 - x - 6$

Graph to show that the quadratic function $f(x) = x^2 - x - 6$ and the function in factored form $f(x) = (x+2)(x-3)$ have the same graph and roots

$y = x^2 - x - 6$



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Build a Polynomial Equation with Two Integers Roots

- Write the factors
- Multiply factors together and simplify
- Check by graphing the equation

$x = 2$ $\quad$ $x = 5$

$(x - 2)$ $\quad$ $(x - 5)$

$(x - 2)(x - 5)$

$x^2 - 5x - 2x + 10$

Combine Like Terms

$x^2 - 5x - 2x + 10$

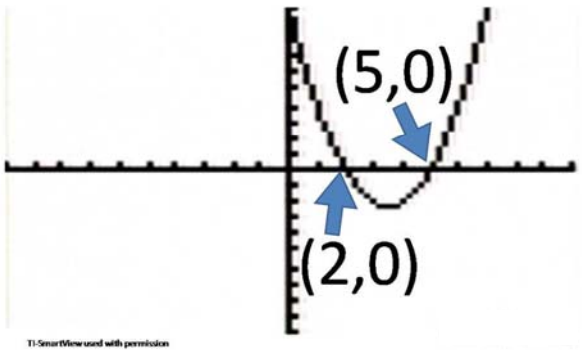
$x^2 - 7x + 10$

Graph

$y = x^2 - 7x + 10$

to verify that the equation has roots at $x = 2$ and $x = 5$

$x^2 - 7x + 10$



Build a Polynomial Equation With Two Integer Roots

Find a polynomial with roots at $x = -3$ and $x = 4$.

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Given the roots

$x = -3$ $x = 4$

the factors are

$(x + 3)$ $(x - 4)$

$x = -3$ $x = 4$

$(x - -3)$ $(x - 4)$

$(x + 3)(x - 4)$

$x = -3$ $x = 4$

$(x - -3)$ $(x - 4)$

$(x + 3)(x - 4)$

$x^2 - 4x + 3x - 12$

Combine Like Terms

$x^2 - 4x + 3x - 12$

$x^2 - x - 12$

Graph

$y = x^2 - x - 12$

to verify that the equation has roots at $x = -3$ and $x = 4$

Build a Polynomial Equation with Three Integer Roots

Find a polynomial with roots at $x = -4$, $x = 2$, and $x = 6$.

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$x = -4 \quad x = 2 \quad x = 6$
$(x + 4) \quad (x - 2) \quad (x - 6)$

$(x + 4)(x - 2)(x - 6)$

$x = -4 \quad x = 2 \quad x = 6$
$(x + 4) \quad (x - 2) \quad (x - 6)$

$(x + 4)(x - 2)(x - 6)$

$(x + 4)(x^2 - 8x + 12)$

$x = -4 \quad x = 2 \quad x = 6$
$(x + 4) \quad (x - 2) \quad (x - 6)$

$(x + 4)(x^2 - 8x + 12)$

$x^3 - 8x^2 + 12x$

$x = -4 \quad x = 2 \quad x = 6$
$(x + 4) \quad (x - 2) \quad (x - 6)$

$(x + 4)(x^2 - 8x + 12)$

$x^3 - 8x^2 + 12x + 4x^2 - 32x + 48$

Combine Like Terms

$x^3 - 8x^2 + 12x + 4x^2 - 32x + 48$

$x^3 - 4x^2 - 20x + 48$

Graph

$y = x^3 - 4x^2 - 20x + 48$

to verify that the equation has roots at $x = -4$, $x = 2$, and $x = 6$

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Build a Polynomial Equation with Irrational Roots

Find a polynomial with roots at $x=-1$, $x=\sqrt{2}$, and $x=-\sqrt{2}$.

$x = -1 \quad x = \sqrt{2} \quad x = -\sqrt{2}$

$(x + 1) \quad (x - \sqrt{2}) \quad (x + \sqrt{2})$

$(x+1)(x-\sqrt{2})(x+\sqrt{2})$

$(x+1)(x^2+\sqrt{2}x-\sqrt{2}x-\sqrt{4})$

$(x+1)(x^2-2)$

$x = -1 \quad x = \sqrt{2} \quad x = -\sqrt{2}$

$(x + 1) \quad (x - \sqrt{2}) \quad (x + \sqrt{2})$

$(x + 1)(x^2 - 2)$

$x^3 - 2x + x^2 - 2$

Reorder terms in decreasing order of exponents:

x^3-2x+x^2-2

$x^3 + x^2 - 2x - 2$

Graph

$y = x^3 + x^2 - 2x - 2$

to verify that the equation has roots at $x = -1$, $x = \sqrt{2}$, and $x = -\sqrt{2}$

Build a Polynomial Equation with a Root at Zero

Find a polynomial with roots at $x = 0$, $x = -1$, and $x = 1$

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$x = 0 \quad x = -1 \quad x = 1$

$(x - 0) \quad (x + 1) \quad (x - 1)$

$x(x + 1)(x - 1)$

$x = 0 \quad x = -1 \quad x = 1$

$(x - 0) \quad (x + 1) \quad (x - 1)$

$x(x + 1)(x - 1)$

$x(x^2 - x + x - 1)$

$x(x^2 - 1)$

$x = 0 \quad x = -1 \quad x = 1$

$(x - 0) \quad (x + 1) \quad (x - 1)$

$x(x^2 - 1)$

$x^3 - x$

Graph

$y = x^3 - x$

to verify that the equation has roots at $x = 0$, $x = -1$, and $x = 1$

Build a Polynomial Equation With a Fraction as a Root

Find a polynomial with roots at $x = 0$, $x = -2$, and $x = \frac{3}{4}$

$x = 0 \quad x = -2 \quad x = \frac{3}{4}$

$x(x + 2)(x - \frac{3}{4})$

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$x = 0 \quad x = -2 \quad x = \frac{3}{4}$

$x(x + 2)(x - \frac{3}{4})$

$x - \frac{3}{4} = 0$

$(4)x - (4)\frac{3}{4} = (4)0$

Multiply by 4 to clear the fraction

$x = 0 \quad x = -2 \quad x = \frac{3}{4}$

$x(x + 2)(x - \frac{3}{4})$

$x - \frac{3}{4} = 0$

$(4)x - (4)\frac{3}{4} = (4)0$

$4x - 3 = 0$

$x = 0 \quad x = -2 \quad x = \frac{3}{4}$

$x(x + 2)(x - \frac{3}{4})$

$x(x + 2)(4x - 3)$

$x = 0 \quad x = -2 \quad x = \frac{3}{4}$

$x(x + 2)(x - \frac{3}{4})$

$x(x + 2)(4x - 3)$

$x(4x^2 - 3x + 8x - 6)$

Combine Like Terms

$x(4x^2 - 3x + 8x - 6)$

$x(4x^2 + 5x - 6)$

$x = 0 \quad x = -2 \quad x = \frac{3}{4}$

$x(4x^2 + 5x - 6)$

$4x^3 + 5x^2 - 6x$

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Graph

$y = 4x^3 + 5x^2 - 6x$

to verify that the equation has roots at $x = 0$, $x = -2$, and $x = \frac{3}{4}$

Find a Polynomial with Roots of

$-\frac{2}{3}$, $\frac{1}{5}$, and 2

$x = -\frac{2}{3}$ $x = \frac{1}{5}$ $x = 2$

$x + \frac{2}{3} = 0$ $x - \frac{1}{5} = 0$

$3x + 2 = 0$ $5x - 1 = 0$

$(3x + 2)$ $(5x - 1)$

$x = -\frac{2}{3}$ $x = \frac{1}{5}$ $x = 2$

$(3x + 2)(5x - 1)(x - 2)$

$x = -\frac{2}{3}$ $x = \frac{1}{5}$ $x = 2$

$(3x + 2)(5x - 1)(x - 2)$

$(3x + 2)(5x^2 - 10x - x + 2)$

Combine Like Terms

$(3x + 2)(5x^2 - 10x - x + 2)$

$(3x + 2)(5x^2 - 11x + 2)$

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$(3x + 2)(5x^2 - 11x + 2)$

$15x^3 - 33x^2 + 6x + 10x^2 - 22x + 4$

Combine Like Terms

$15x^3 - 33x^2 + 6x + 10x^2 - 22x + 4$

$15x^3 - 23x^2 - 16x + 4$

Combine Like Terms

$15x^3 - 33x^2 + 6x + 10x^2 - 22x + 4$

$15x^3 - 23x^2 - 16x + 4$

Graph

$y = 15x^3 - 23x^2 - 16x + 4$

to verify that the equation has roots at $x = -\frac{2}{3}$, $x = \frac{1}{5}$, and $x = 2$