Slides from Solving Systems of Linear Equations (Tutorial 15)

Solving Systems of Linear Equations

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This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

Goal

Solve systems of linear equations using three methods:

- Graphing
- Substitution
- Elimination

A linear system consists of two linear equations identified with a bracket

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

To solve a system, find one ordered pair that satisfies both equations.

Graphing Method

- Put the equations in "y =" format to enter into the calculator
- Graph the equations and observe the point of intersection
- Verify that the point of intersection is the solution to each equation in the system

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

Put the equations in "y =" format

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

$$3x + 2y = 8$$

$$\cancel{3x} + 2y - \cancel{3x} = 8 - 3x$$

$$2y = 8 - 3x$$

$$y = \frac{8 - 3x}{2}$$

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$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

$$2x + y = 5$$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

$$2x + y = 5$$

$$2x + y - 2x = 5 - 2x$$

$$y = 5 - 2x$$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

System of equations in "y =" format

$$\begin{cases} y = \dfrac{8 - 3x}{2} \\ y = 5 - 2x \end{cases}$$

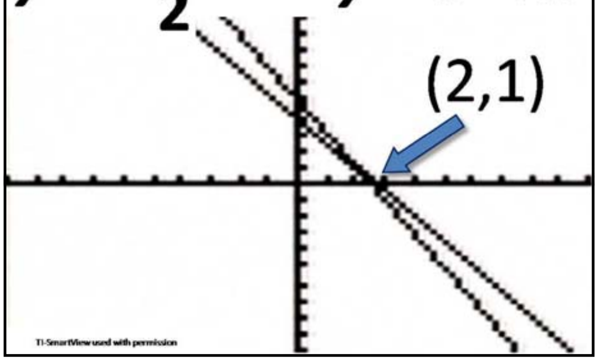
Graph The Equations Using A Calculator

$$y = \frac{8 - 3x}{2}$$

$$y = 5 - 2x$$

Observe the point of intersection

$$y = \frac{8 - 3x}{2} \qquad y = 5 - 2x$$



$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases} \qquad (2,1)$$

Verify the solution by substituting the ordered pair into both equations

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$$\begin{cases} \boxed{3x + 2y = 8} \\ 2x + y = 5 \end{cases} \quad (2,1)$$

$$3(2) + 2(1) = 8$$
$$6 + 2 = 8$$
$$8 = 8 \checkmark$$

$$\begin{cases} 3x + 2y = 8 \\ \boxed{2x + y = 5} \end{cases} \quad (2,1)$$

$$2(2) + 1 = 5$$
$$4 + 1 = 5$$
$$5 = 5 \checkmark$$

Substitution Method

- Solve one equation for one variable
- Substitute the resulting expression for that variable in the other equation and solve for the remaining variable
- Substitute the value of the known variable into an equation to find the value of the second variable
- Verify the solution

$$\begin{cases} 3x + 2y = 8 \\ \boxed{2x + y = 5} \end{cases}$$

Solve one equation for one variable

$$2x + y = 5$$
$$y = 5 - 2x$$

$$\begin{cases} 3x + 2\boxed{y} = 8 \\ 2x + y = 5 \end{cases}$$

Substitute resulting expression into the other equation and solve for the second variable

$$y = \boxed{5 - 2x}$$

$$\begin{cases} \boxed{3x + 2y = 8} \\ 2x + y = 5 \end{cases}$$

$$3x + 2(5 - 2x) = 8$$
$$3x + 10 - 4x = 8$$
$$10 - x = 8$$
$$-x = -2$$
$$x = 2$$

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$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

Find the value of the second variable

$y = 5 - 2x$
$y = 5 - 2(2)$
$y = 5 - 4$
$y = 1$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases} \quad (2,1)$$

Verify the solution

Elimination Method

- Multiply one (or both) of the equations so one term is the opposite of a term in the other
- Add equations to eliminate one variable
- Substitute the value of the known variable into an equation to find the value of the second variable
- Verify the solution

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

Multiple one equation so the y-terms are opposites

$-2(2x + y = 5)$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

Multiple one equation so the y-terms are opposites

$-2(2x + y = 5)$

$-4x - 2y = -10$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

Add to eliminate the y-terms

$3x + 2y = 8$
$-4x - 2y = -10$

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$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

$$\begin{array}{r} 3x + 2y = 8 \\ \underline{-4x - 2y = -10} \\ -x \quad\quad = -2 \\ x = 2 \end{array}$$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases} \quad (2,\ )$$

Substitute the value of x to find the value of y

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases} \quad (2,1)$$

$$\begin{aligned} 2(2) + y &= 5 \\ 4 + y &= 5 \\ y &= 5 - 4 \\ y &= 1 \end{aligned}$$

$$\begin{cases} 3x + 2y = 8 \\ 2x + y = 5 \end{cases} \quad (2,1)$$

Verify the solution

First Special Case

✓ ***What happens when the graphs of the equations in a system are parallel?***

Parallel Lines

$$\begin{cases} y = 2x - 3 \\ y = 2x + 2 \end{cases}$$

- The equations are in slope-intercept form
- Both have a slope of 2
- The lines are parallel and will never intersect

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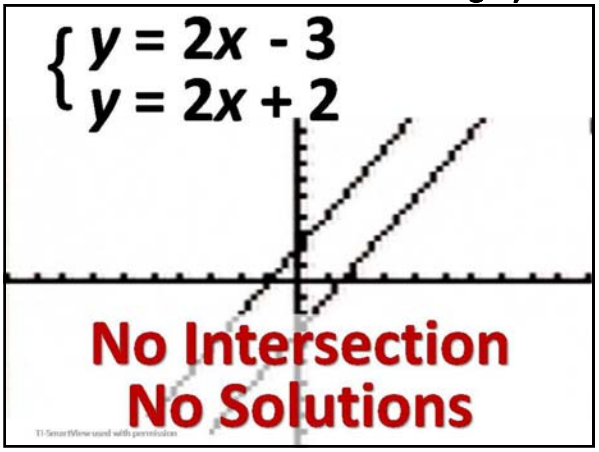

✓ Apply Substitution Method

$$\begin{cases} y = 2x - 3 \\ y = 2x + 2 \end{cases}$$

$$\begin{cases} y = 2x - 3 \\ y = \boxed{2x + 2} \end{cases}$$

$$\begin{array}{rcl} 2x + 2 & = & 2x - 3 \\ -2x & & -2x \\ \hline 2 & \not= & -3 \end{array}$$

False Statement
No Solutions

Second Special Case

✓ *What happens when the two equations in a system represent the same line?*

Same Line

$$\begin{cases} x - y = 4 \\ 3x - 3y = 12 \end{cases}$$

If these two equations are graphed, there will only be one line

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$$\begin{cases} x - y = 4 \\ 3x - 3y = 12 \end{cases}$$

- ✓ Confirm Same Line
- ✓ Infinite Number of Solutions
- ✓ Apply the Elimination Method

$$\begin{cases} -3(x - y = 4) \\ 3x - 3y = 12 \end{cases}$$

$$\begin{cases} -3(x - y = 4) \\ 3x - 3y = 12 \end{cases}$$

$$\begin{aligned} -3x + 3y &= -12 \\ 3x - 3y &= 12 \\ \hline \end{aligned}$$

$$\begin{cases} -3(x - y = 4) \\ 3x - 3y = 12 \end{cases}$$

$$\begin{aligned} -3x + 3y &= -12 \\ 3x - 3y &= 12 \\ \hline 0 &= 0 \end{aligned}$$

$$\begin{cases} x - y = 4 \\ 3x - 3y = 12 \end{cases}$$

Since, 0 = 0 is a true statement

- The two equations define the same line
- There are infinitely many solutions to the linear system
- Every point on that line is a solution to the linear system

Application

How much pure alcohol do I need to add to 2 liters (ℓ) of a 30% alcohol solution so that I'll have a 65% alcohol solution?

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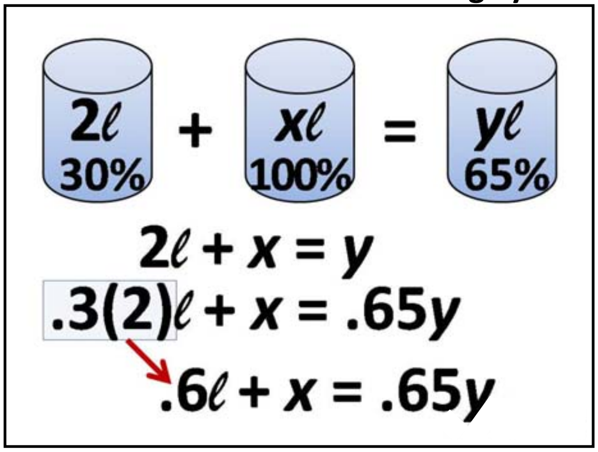

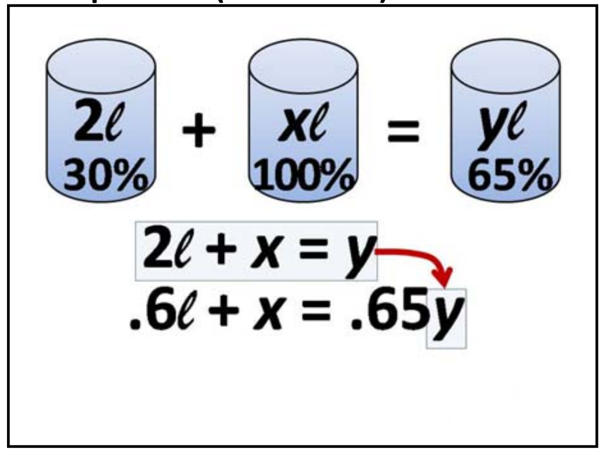

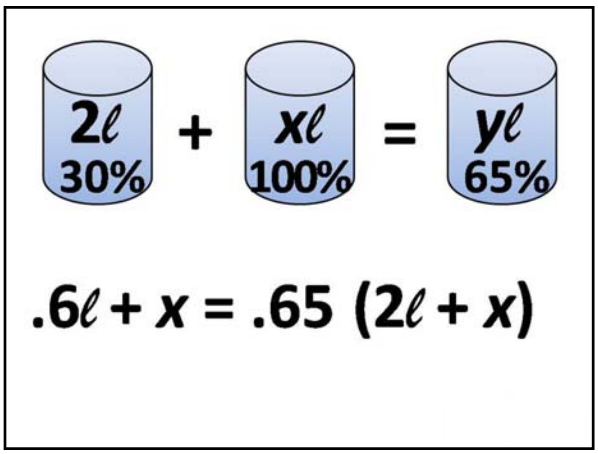

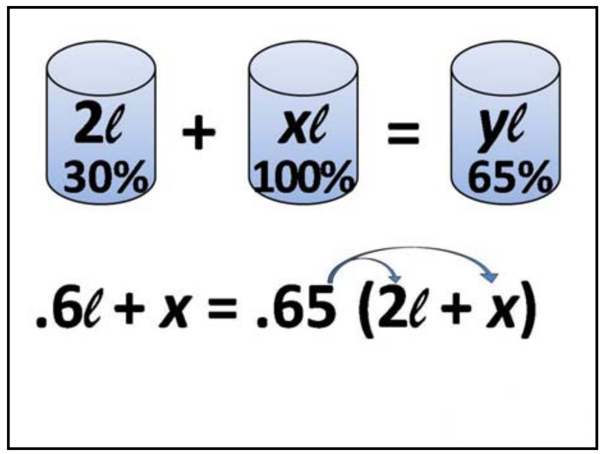

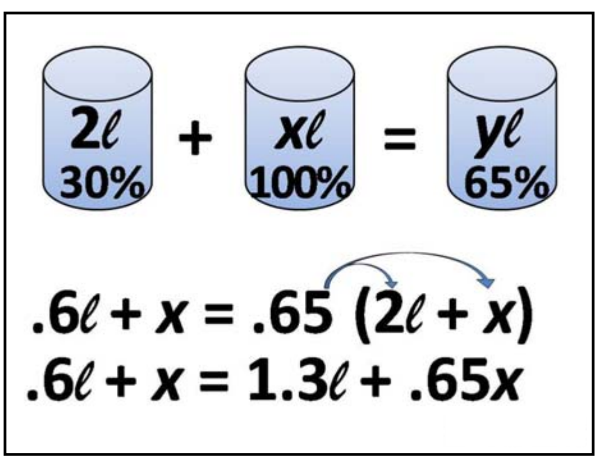

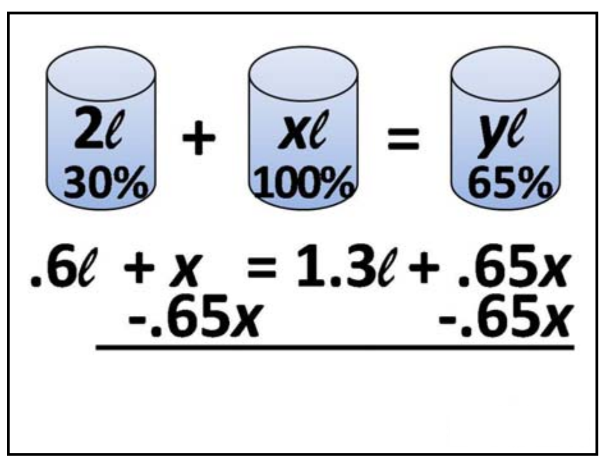

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