Slides from Exponents (Tutorial 20)

Exponents:
Meaning and Laws

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This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

- **An exponent indicates the number of times that a number and/or variable is multiplied by itself**
- **An exponent may also be called a power**
- **A variable raised to a power is the base**

Example:

$$2^3 = (2)(2)(2) = 8$$

$$x^4 = x \cdot x \cdot x \cdot x$$

Raise 2*x* to the third power means

$$(2x)^3 = (2x)(2x)(2x) = 8x^3$$

Which is the same as

$$(2x)^3 = 2^3 \cdot x^3 = 8x^3$$

Note: When $2x^3$ is written without the parenthesis, the 2 does not get raised to the third power

Laws for Exponents

Multiplication Law: When multiplying numbers with like bases raised to powers, multiple the coefficients and add the exponents

$$x^3 \cdot x^2 = x{\cdot}x{\cdot}x{\cdot}x{\cdot}x = x^5$$

OR

$$x^3 \cdot x^2 = x^{3+2} = x^5$$

$$a^4 \cdot a^5 = a{\cdot}a{\cdot}a{\cdot}a{\cdot}a{\cdot}a{\cdot}a{\cdot}a{\cdot}a = a^9$$

OR

$$a^4 \cdot a^5 = \mathrm{a}^{4+5} = a^9$$

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Multiplication Law: When multiplying numbers with like bases raised to powers, multiple the coefficients and add the exponents

$$2x^3 \cdot 4x^5 =$$
$$2 \cdot x \cdot x \cdot x \cdot 4 \cdot x \cdot x \cdot x \cdot x \cdot x = 8x^8$$

OR

$$2x^3 \cdot 4x^5 = 8x^{3+5} = 8x^8$$

Division Law: When dividing like bases raised to powers, divide the coefficients and subtract the exponents

$$\frac{a^6}{a^3} = \frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a} = a^3$$

Division Law: When dividing like bases raised to powers, divide the coefficients and subtract the exponents

$$\frac{m^5}{m^2} = \frac{m \cdot m \cdot m \cdot m \cdot m}{m \cdot m} = m^3$$

Division Law: When dividing like bases raised to powers, divide the coefficients and subtract the exponents

$$\frac{4x^7}{2x^2} = \frac{4 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot x \cdot x} = 2x^5$$

Division Law: When dividing like bases raised to powers, divide the coefficients and subtract the exponents

$$\frac{4x^7}{2x^2} = \frac{\textcircled{4} \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot x}{\textcircled{2} \cancel{x} \cdot \cancel{x}} = 2x^5$$

4 ÷ 2 = 2

Raising a "Power of a Power"

1. **Apply the Multiplication Law**

$$(n^2)^3 = (n^2)(n^2)(n^2) = (n \cdot n)(n \cdot n)(n \cdot n) = n^6$$

$$(a^3)^5 = (a^3)(a^3)(a^3)(a^3)(a^3) = a^{15}$$

$$(4x^3)^2 = (4x^3)(4x^3) = 16x^6$$

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2. Or find a "Power of a Power"

Power of a Power Law: When raising a base with an exponent (power) to a power, raise the coefficient to the power and multiply the exponents

$$(4x^3)^2 = 4^2 \cdot (x^3)^2 = 16x^6$$

Negative Exponent Law: When an exponent is negative, the base of the exponent can be rewritten as one divided by the base raised to the positive exponent

Note: Since $a^0 = 1$, substitute a^0 for 1 and apply the division law

$$\frac{a^3}{a^5} = \frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a \cdot a} = \frac{1}{a^2} = \frac{a^0}{a^2} = a^{-2}$$

Negative Exponent Law: When an exponent is negative, the base of the exponent can be rewritten as one divided by the base raised to the positive exponent

$$\frac{x^4}{x^8} = x^{-4} = \frac{1}{x^4}$$

Negative Exponent Law: When an exponent is negative, the base of the exponent can be rewritten as one divided by the base raised to the positive exponent

$$x^{-2} = \frac{1}{x^2}$$

When adding or subtracting terms with exponents, look for like terms, meaning having like variables raised to the same exponents, then add or subtract the coefficients

$$2x^2 + 3x^2 = 5x^2$$

When adding or subtracting terms with exponents, look for like terms, meaning having like variables raised to the same exponents, then add or subtract the coefficients

$$2x^2 + 3x^4$$

Note: $2x^2 + 3x^4$ can not be added because the exponents of x are different

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When multiplying terms with exponents, multiply the coefficients and add the exponents of the like bases

$$x^2y^3 \cdot (xy^2)^4 = x^2y^3 \cdot x^4y^8$$
$$= x^{2+4}y^{3+8}$$
$$= x^6y^{11}$$

Division Law: When dividing numbers with like bases raised to powers, divide the coefficients and subtract the exponents

$$\frac{m^5}{m^2} = \frac{m \cdot m \cdot m \cdot m \cdot m}{m \cdot m}$$

When dividing terms with exponents, divide the coefficients and subtract the exponents of like bases

$$\frac{(3x^2y)^3}{3xy} = \frac{27x^6y^3}{3xy}$$

$$= 9x^{6-1}y^{3-1} = 9x^5y^2$$

Application

The diameter of a red blood cell is approximately 7.72 x 10^{-7} meters

Find the area of one cell

$$A = \pi r^2$$

The radius is approximately equal to 3.86 x 10^{-7}*m*

$$A \approx \pi(3.86 \text{ x } 10^{-7}\text{m})^2$$

The diameter of a red blood cell is approximately 7.72 x 10^{-7} meters

Find the area of one cell

$$A \approx \pi(3.86 \text{ x } 10^{-7}\text{m})^2$$
$$A \approx \pi(3.86)^2 \cdot (10^{-7}\text{ m})^2$$
$$A \approx \pi(3.86)^2 \cdot (10^{-14}\text{ m}^2)$$

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The diameter of a red blood cell is approximately 7.72 x 10^{-7} meters

Find the area of one cell

$$A \approx 46.8084739 \cdot 10^{-14} \text{m}^2$$

$$A \approx 4.68084739 \cdot 10^{-13} \text{m}^2$$

$$A \approx 4.68 \cdot 10^{-13} \text{m}^2$$

Expressed in Scientific Notation

The diameter of a red blood cell is approximately 7.72 x 10^{-7} meters

Find the area of one cell

$$A \approx 4.68084739 \cdot 10^{-13} \text{m}^2$$

So, the area of one red blood cell is approximately $4.68084739 \cdot 10^{-13}$ meters^2

Simplify a Binomial with Negative Exponents

$$(a^{-1}+b^{-1})^{-1} = \left(\frac{1}{a}+\frac{1}{b}\right)^{-1}$$

Rewrite the two terms in the given binomial with positive exponents

Simplify a Binomial with Negative Exponents

$$(a^{-1}+b^{-1})^{-1} = \left(\frac{1}{a}+\frac{1}{b}\right)^{-1} = \left(\frac{1}{a}\left[\frac{b}{b}\right]+\frac{1}{b}\left[\frac{a}{a}\right]\right)^{-1}$$

Multiply the first fraction by "*b* over *b*" which is equal to the multiplicative identity 1 and the second fraction by "*a* over *a*" which is also equal to the multiplicative identity 1

Simplify a Binomial with Negative Exponents

$$(a^{-1}+b^{-1})^{-1} = \left(\frac{1}{a}+\frac{1}{b}\right)^{-1} = \left(\frac{1}{a}\left[\frac{b}{b}\right]+\frac{1}{b}\left[\frac{a}{a}\right]\right)^{-1}$$

$$= \left(\frac{b}{ab}+\frac{a}{ab}\right)^{-1}$$

Combine terms because they now have the same denominator

Simplify a Binomial with Negative Exponents

$$(a^{-1}+b^{-1})^{-1} = \left(\frac{1}{a}+\frac{1}{b}\right)^{-1} = \left(\frac{1}{a}\left[\frac{b}{b}\right]+\frac{1}{b}\left[\frac{a}{a}\right]\right)^{-1}$$

$$= \left(\frac{b}{ab}+\frac{a}{ab}\right)^{-1} = \left(\frac{a+b}{ab}\right)^{-1}$$

Rewrite the quantity with a positive exponent

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Simplify a Binomial with Negative Exponents

$$(a^{-1}+b^{-1})^{-1} = \left(\frac{1}{a}+\frac{1}{b}\right)^{-1} = \left(\frac{1}{a}\left[\frac{b}{b}\right]+\frac{1}{b}\left[\frac{a}{a}\right]\right)^{-1}$$

$$= \left(\frac{b}{ab}+\frac{a}{ab}\right)^{-1} = \left(\frac{a+b}{ab}\right)^{-1} = \frac{ab}{a+b}$$

A Reminder: When Squaring Binomials

Remember:

$$(a+b)^2 = (a+b)(a+b)$$

A Reminder: When Squaring Binomials

Remember:

$$(a+b)^2 = (a+b)(a+b)$$

Using FOIL:

$$(a+b)(a+b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$