Tutorials for High School Mathematics

Slides from Right Triangle Trig (Tutorial 22)

Right Triangle Trig

Trigonometric functions, the sine, the cosine, and the tangent

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This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

Right Triangle Trig

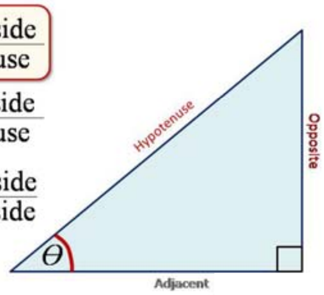
- Trigonometry involves finding and using the lengths of the sides and the measures of the angles in right triangles.
- Trig is used in physics or engineering for finding things such as velocity, torque, and acceleration.

Definition of the trigonometric functions for sine, cosine, and tangent in a right triangle.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



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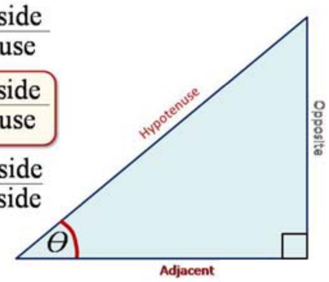

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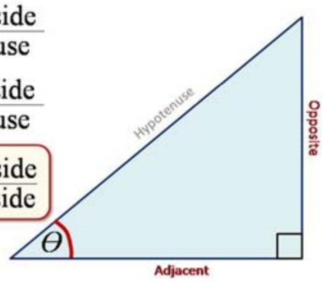

There are three other trigonometric functions,

- **the secant** (sec),
- **the cosecant** (csc), **and**
- **the cotangent** (cot or ctn)

which are the inverses of the more common trig functions.

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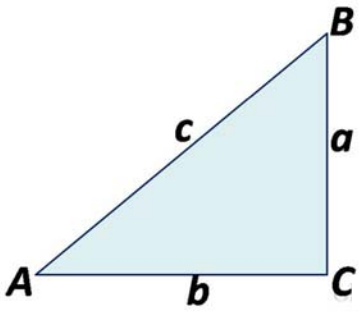
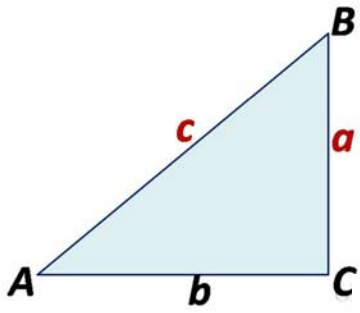
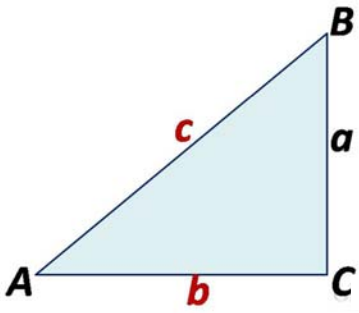
In a triangle, angles are labeled with capital letters and the sides are labeled with lower case letters.



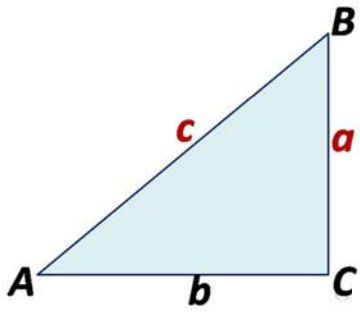
$\sin A = \frac{a}{c}$



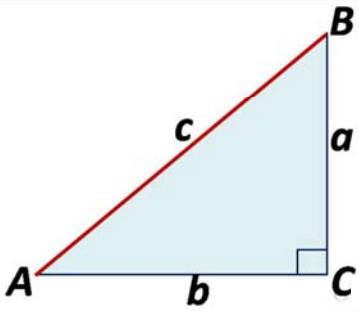
$\sin B = \frac{b}{c}$



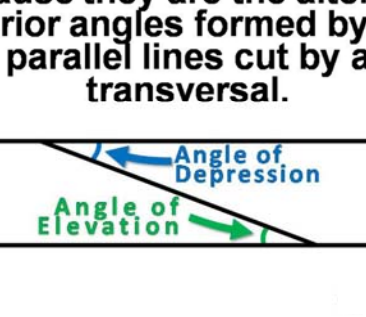
$\cos B = \frac{a}{c}$



✓ Remember the angle under discussion determines the opposite and adjacent sides. In a right triangle, the side opposite the 90° angle is always the hypotenuse.



The angle of elevation and the angle of depression are equal because they are the alternate interior angles formed by two parallel lines cut by a transversal.



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A plane is flying at 2000 feet with an angle of depression to the airport of 23°.

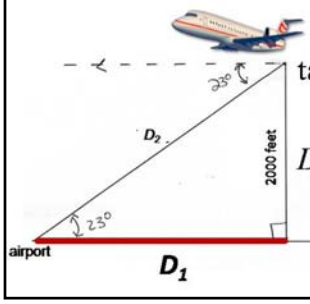
Since the angle of elevation forms a triangle with the missing information we can use the trig functions to find the missing information.

There are two questions

Question 1:
How far is the plane from the airport measured on land?

Question 2:
How far does the plane have to fly in order to land at the airport?

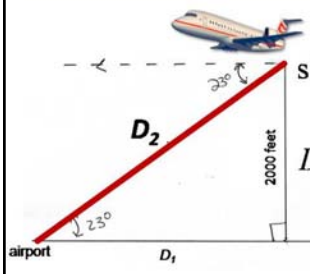
Question 1:
How far is the plane from the airport (D_1)?



$$\tan 23° = \frac{2000'}{D_1}$$

$$D_1 = \frac{2000'}{\tan 23°} \approx 4712 \text{ feet}$$

Question 2:
How far does the plane have to fly in order to land at the airport (D_2)?



$$\sin 23° = \frac{2000'}{D_2}$$

$$D_2 = \frac{2000'}{\sin 23°} \approx 5119 \text{ feet}$$

In another problem, a pole stands in a parking lot.

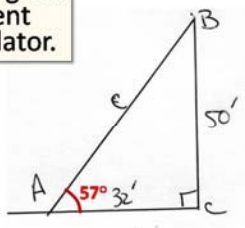
The pole is 50 feet high.

A guy-wire is anchored 32 feet from the base of the pole.

Find the length of the guy-wire.

Find the length of the guy-wire.

✓ Find the measure of angle A using the inverse tangent function on your calculator.



$$\tan A = \frac{50 \text{ feet}}{32 \text{ feet}}$$

$$\tan^{-1}\left(\frac{50'}{32'}\right) = \arctan\left(\frac{50'}{32'}\right) \approx 57.38°$$

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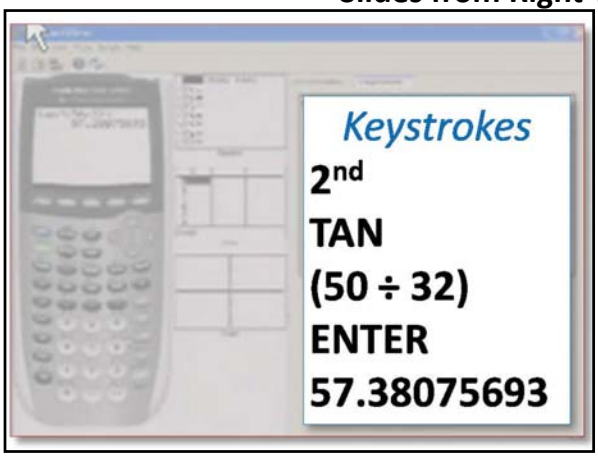
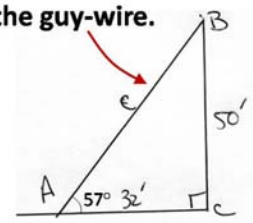


✓Complete the problem on your own

Find the length of the guy-wire.



Results

$$\cos A = \frac{32 \text{ feet}}{c}$$

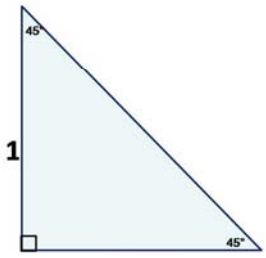
$$c = \frac{32'}{\cos A} = \frac{32 \text{ feet}}{\cos 57.380} \approx 59.36 \text{ feet}$$

Trig Values for Special Angles

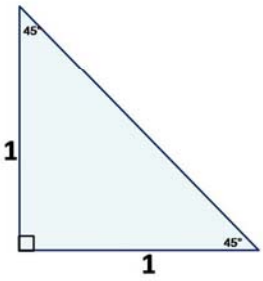
Our first special triangle is the 45°- 45°- 90° triangle

Let's find common values for the lengths of the sides

Assume that one leg of the triangle has length 1



Since the angles are congruent, the other leg has length 1



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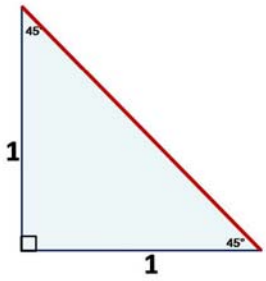
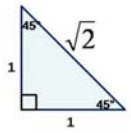
Use the Pythagorean theorem to find the length of the hypotenuse



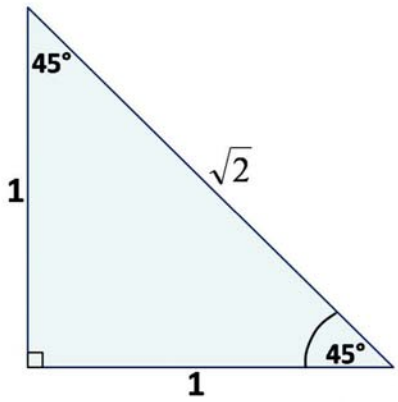


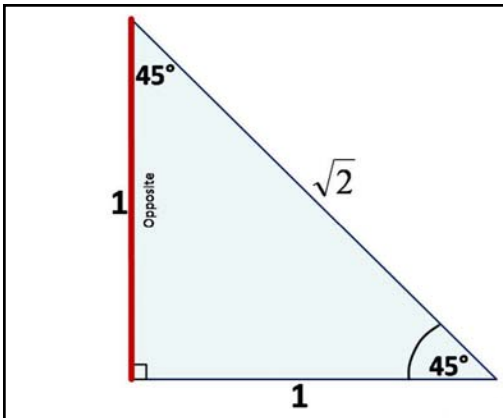
$1^2 + 1^2 = c^2$

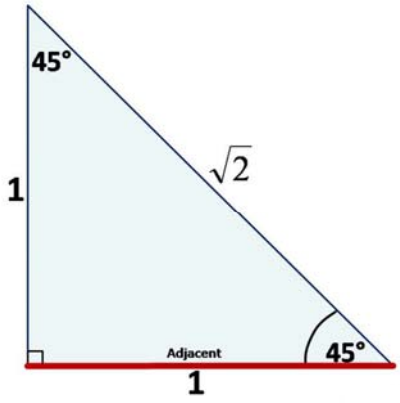
$1 + 1 = c^2$

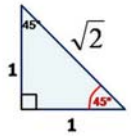
$2 = c^2$

$\sqrt{2} = c$









$\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° =$

$\tan 45° =$

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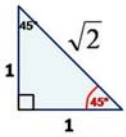
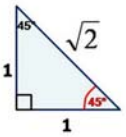


$$\sin 45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° =$$

$$\tan 45° =$$



$$\sin 45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

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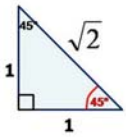


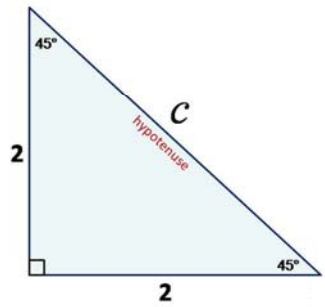
$$\sin 45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{1}{1} = 1$$

Let's find out what happens with a larger or smaller 45° – 45° – 90° degree triangle.

Suppose the two equal legs are 2



✓**Remember: trig values are RATIOS of the sides and triangles are proportionally similar, the sine, cosine, and tangent of an angle will always be the same no matter how big we draw the triangle.**

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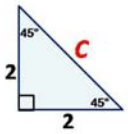
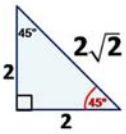


$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$8 = c^2$$
$$\sqrt{8} = c$$
$$\sqrt{4} \cdot \sqrt{2} = c$$
$$2\sqrt{2} = c$$

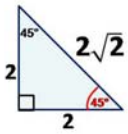


Let's find sine, cosine and tangent using this triangle.

$$\sin 45° = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\cos 45° =$

$\tan 45° =$

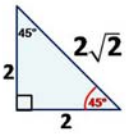


Let's find sine, cosine and tangent using this triangle.

$$\sin 45° = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\tan 45° =$



Let's find sine, cosine and tangent using this triangle.

$$\sin 45° = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

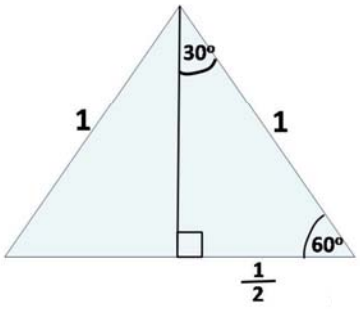
$$\cos 45° = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{2}{2} = 1$$

Our next special triangle is the 30°- 60°- 90° triangle.

Let's find common values for the lengths of the sides.

Let's find common values for the lengths of the sides in a 30°- 60°- 90° triangle.



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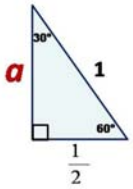


$$a^2 + \left(\frac{1}{2}\right)^2 = 1^2$$

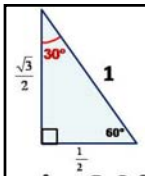
$$a^2 + \frac{1}{4} = 1$$

$$a^2 = \frac{3}{4}$$

$$a = \sqrt{\frac{3}{4}}$$

$$a = \frac{\sqrt{3}}{2}$$

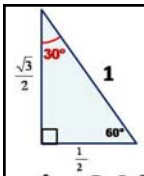
Now let's find the values of sine, cosine, and tangent for the angles in our 30°– 60°– 90° triangle.



$\sin 30° = \frac{1}{2}$

$\cos 30° =$

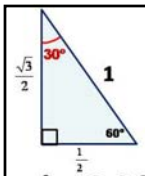
$\tan 30° =$



$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

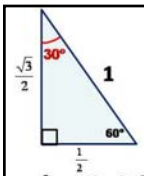
$\tan 30° =$



$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$



$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

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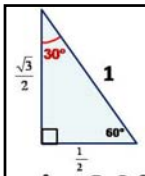
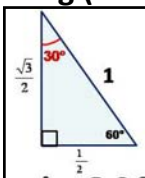


$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

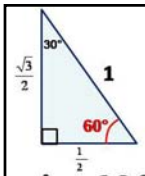
$$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$



$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

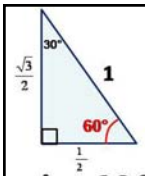
$$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° =$$

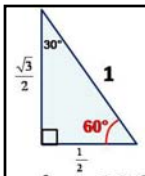
$$\tan 60° =$$



$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$$\tan 60° =$$



$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$$\tan 60° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

Summary

Today we learned about…

- **Right triangle trigonometry**
- **Two special triangles and the sine, cosine, and tangent for their angles.**