

Slides from The Laws of Sines and Cosines (Tutorial 23)

The Laws of Sines and Cosines

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Paul Lenz

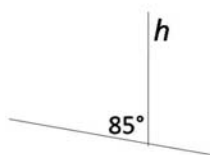
This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

Objectives

- Learn to use the law of sines and the law of cosines
- We usually think of right triangles in trigonometry
- The law of sines and the law of cosines allow us to use trigonometry to find parts of any triangle

Example 1 Law of Sines

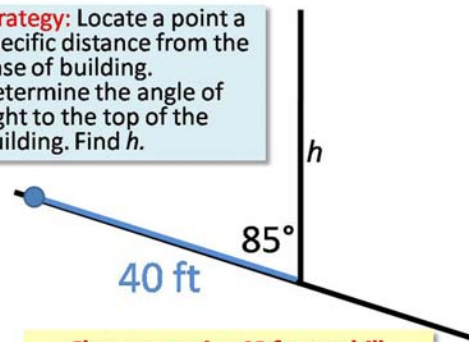
Find the height (h) of a building on a plot of land with a slight incline. The building makes an 85° angle with the ground.



The building is too tall to measure directly. Use indirect measurement.

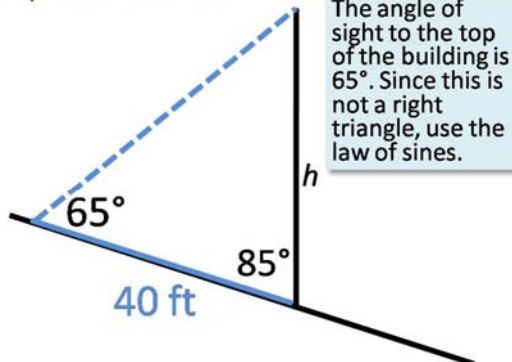
Example 1 Law of Sines

Strategy: Locate a point a specific distance from the base of building. Determine the angle of sight to the top of the building. Find h .



Choose a point 40 feet uphill.

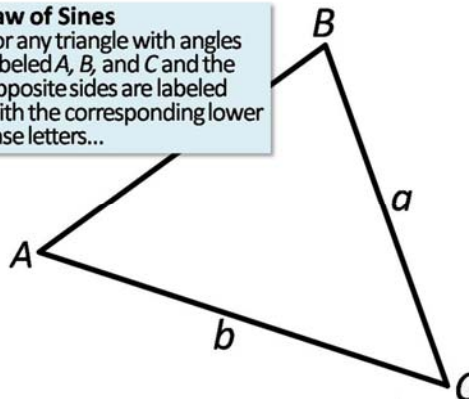
Example 1 Law of Sines



The angle of sight to the top of the building is 65° . Since this is not a right triangle, use the law of sines.

Law of Sines

For any triangle with angles labeled A , B , and C and the opposite sides are labeled with the corresponding lower case letters...



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Law of Sines

For any triangle with angles labeled A, B, and C and the opposite sides labeled with the corresponding lower case letters, the following ratios are equal.

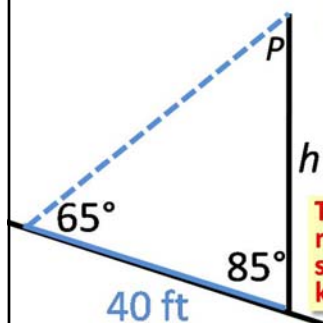
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Applying the Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- Use 2 ratios to find a missing part of a triangle
- Must know values for an angle and its opposite side along with the part opposite the missing part

Example 1 Law of Sines

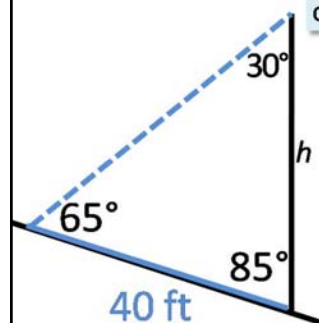


Find the height (h) of the building

$$\frac{\sin 65^\circ}{h}$$

To get the second ratio needed for the law of sines, find angle P with known opposite side.

Example 1 Law of Sines



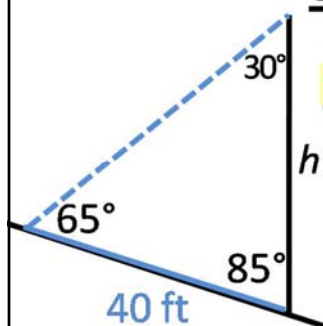
Sum of all three angles of a triangle is 180°

$$P + 65^\circ + 85^\circ = 180^\circ$$

$$P = 180^\circ - 65^\circ - 85^\circ$$

$$P = 30^\circ$$

Example 1 Law of Sines



$$\frac{\sin 65^\circ}{h} = \frac{\sin 30^\circ}{40}$$

Solve proportion for h

Example 1 Law of Sines

$$\frac{\sin 65^\circ}{h} = \frac{\sin 30^\circ}{40}$$

Multiply both sides by h and then by 40

$$(\sin 65^\circ) 40 = (\sin 30^\circ) h$$

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Example 1 Law of Sines

$$\frac{\sin 65^\circ}{h} = \frac{\sin 30^\circ}{40}$$

Multiply both sides by h and then by 40

$$(\sin 65^\circ) 40 = (\sin 30^\circ) h$$

Divide both sides by $\sin 30^\circ$

$$\frac{(\sin 65^\circ) 40}{\sin 30^\circ} = \frac{(\sin 30^\circ) h}{\sin 30^\circ}$$

Example 1 Law of Sines

$$\frac{\sin 65^\circ}{h} = \frac{\sin 30^\circ}{40}$$

Multiply both sides by h and then by 40

$$(\sin 65^\circ) 40 = (\sin 30^\circ) h$$

Divide both sides by $\sin 30^\circ$

$$\frac{(\sin 65^\circ) 40}{\sin 30^\circ} = \frac{\cancel{(\sin 30^\circ)} h}{\cancel{\sin 30^\circ}}$$

Example 1 Law of Sines

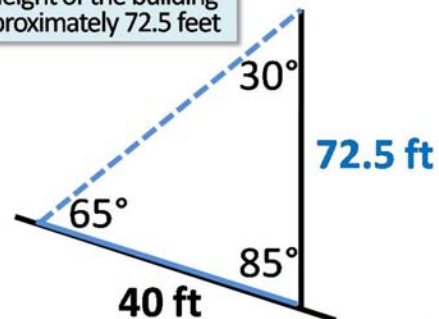
$$\frac{(\sin 65^\circ) 40}{\sin 30^\circ} = h$$

Evaluate using your calculator.
Make sure calculator is in degree mode.

$$72.5 \approx h$$

Example 1 Law of Sines

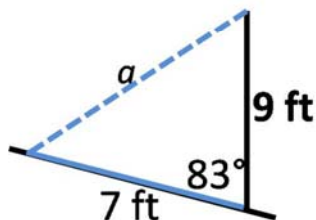
The height of the building is approximately 72.5 feet



Example 2 Law of Cosines

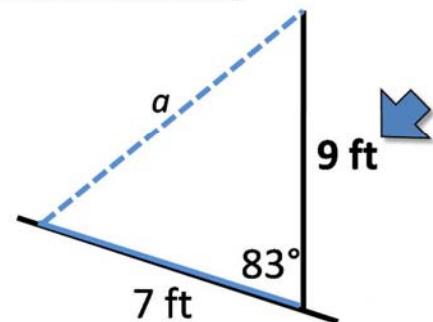
Install a support beam (a) to hold up a 9 foot sign post on a hill.

How long should the support beam be?



What we know:

- Sign post is 9 ft tall

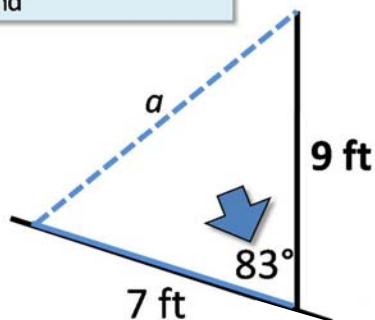


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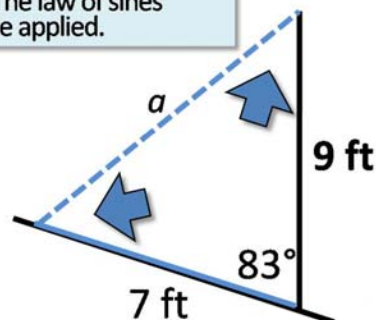
What we know:

- Sign post makes an 83° with the ground



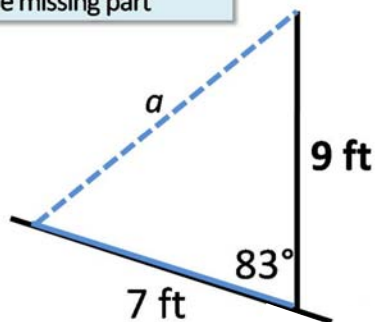
What we know:

- No pairs of opposite parts are known. The law of sines cannot be applied.



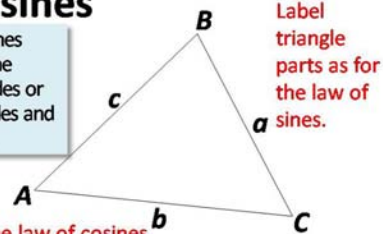
What we know:

- Can apply the law of cosines to find the missing part



Law of Cosines

Apply the law of cosines when, in a triangle, the lengths of all three sides or the lengths of two sides and an angle are known.



Label triangle parts as for the law of sines.

Three forms for the law of cosines

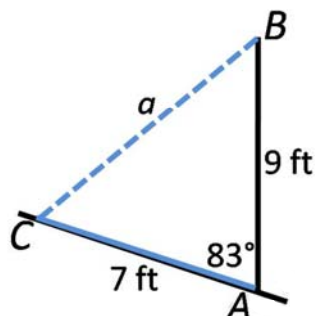
$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Example 2 Law of Cosines

How long should the support beam be?

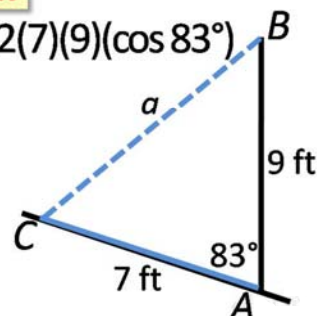


Example 2 Law of Cosines

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Substitute values

$$a^2 = 7^2 + 9^2 - 2(7)(9)(\cos 83^\circ)$$



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Example 2 Law of Cosines

$$a^2 = 7^2 + 9^2 - 2(7)(9)(\cos 83^\circ)$$

Multiply, evaluate cosine, and simplify

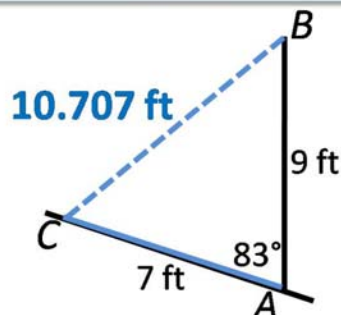
$$a^2 = 49 + 81 - 15.356$$

$$a^2 \approx 114.644$$

$$a \approx 10.707 \text{ ft}$$

Example 2 Law of Cosines

Support beam must be approximately 10.707 ft



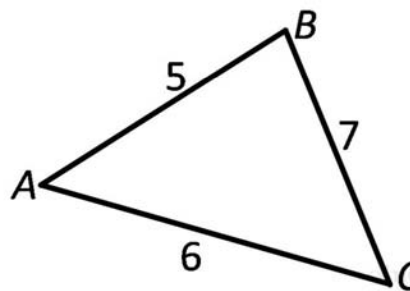
Solving Triangles

To solve a triangle:

- Find all of the missing parts of the given triangle
- Use the law of sines, the law of cosines, and other mathematics formulas as needed.

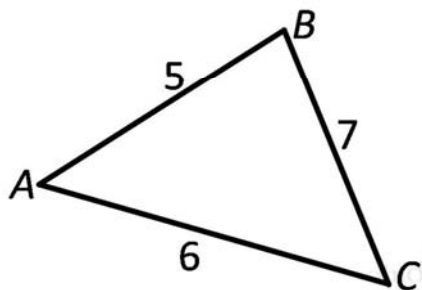
Example 3 Solving Triangles

Find the measures of angles A, B, and C



Example 3 Solving Triangles

Given the lengths of the three sides, apply one of the law of cosines formulas to find an angle.

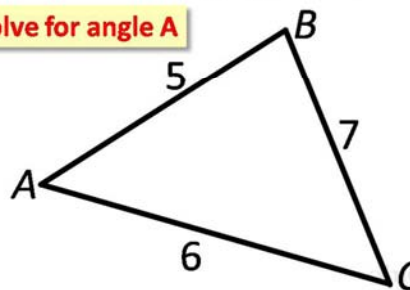


Example 3 Solving Triangles

$$a^2 = b^2 + c^2 - 2bc (\cos A)$$

$$7^2 = 5^2 + 6^2 - 2(5)(6)(\cos A)$$

Solve for angle A



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Example 3 Solving Triangles

$$7^2 = 5^2 + 6^2 - 2(5)(6)(\cos A)$$

$$49 = 25 + 36 - 60(\cos A)$$

60 is the coefficient of the cosine function, so 36 minus 60 cannot be calculated

Example 3 Solving Triangles

$$7^2 = 5^2 + 6^2 - 2(5)(6)(\cos A)$$

$$49 = 25 + 36 - 60(\cos A)$$

$$49 = 61 - 60(\cos A)$$

$$-12 = -60(\cos A)$$

$$0.2 = \cos A$$

cos A is multiplied by -60, so undo by dividing both sides of the equation by -60.

Example 3 Solving Triangles

$$0.2 = \cos A$$

$$\cos^{-1}(0.2) \approx A$$

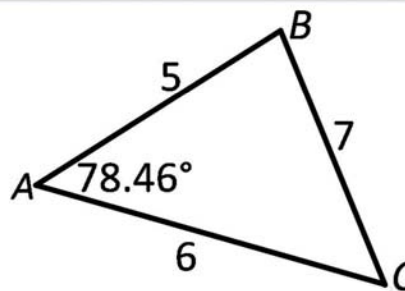
$$78.46^\circ \approx A$$

To solve for A, find the inverse cosine. Use 2nd cos on your calculator.

Add to the diagram

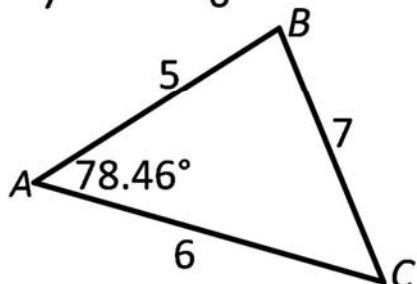
Example 3 Solving Triangles

Now, knowing one angle of the triangle, use the law of sines to find another angle.



Example 3 Solving Triangles

$$\frac{\sin 78.46^\circ}{7} = \frac{\sin B}{6}$$



Example 3 Solving Triangles

$$\frac{\sin 78.46^\circ}{7} = \frac{\sin B}{6}$$

Multiply both sides by 6

$$\frac{(\sin 78.46^\circ)6}{7} = \sin B$$

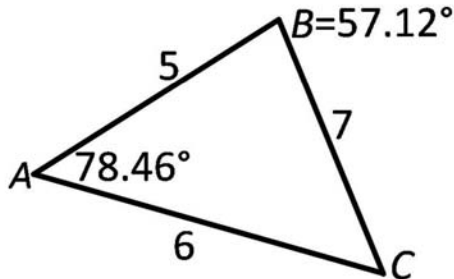
$$.8398 \approx \sin B$$

$$57.12^\circ \approx B$$

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Example 3 Solving Triangles

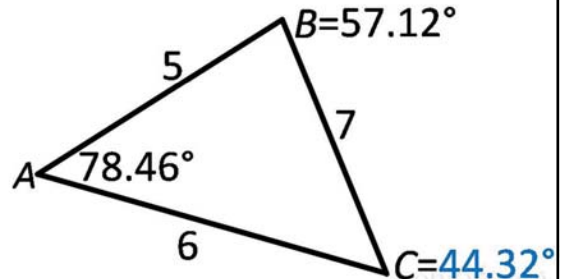
Now, find C



Example 3 Solving Triangles

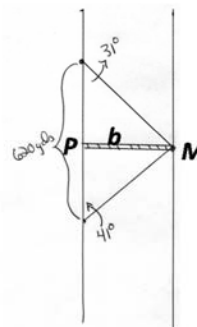
$$180^\circ - 78.46^\circ - 57.12^\circ = m\angle C$$

$$44.42^\circ = m\angle C$$



Two Application Problems

Building a Bridge



Two surveyors stand on a river bank 620 yards apart.

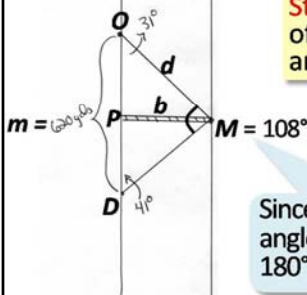
They sight a marker (M) on the other side of the river at angles of 31° and 41° .

From M , a bridge will be constructed perpendicular to the other river bank.

What will be the length (b) of a bridge across the river?

Building a Bridge

What will be the length (b) of the bridge across the river?



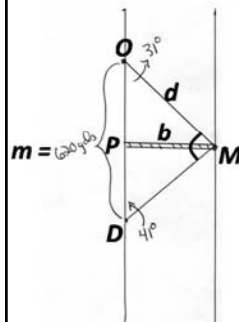
Strategy: Apply the law of sines to find d . Use d and $\sin 31^\circ$ to find b .

Since the sum of all three angles of triangle OMD is 180° , angle M is 108° .

Building a Bridge

Step 1:

In triangle OMD , apply the law of sines to find the length of side d .



$$\frac{\sin D}{d} = \frac{\sin M}{m}$$

$$\frac{\sin 41^\circ}{d} = \frac{\sin 108^\circ}{620}$$

$$\frac{(\sin 41^\circ \cdot 620)}{\sin 108^\circ} = d$$

$$427.68 \approx d$$

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Building a Bridge
Step 1:

Save the value of d (427.68) on your calculator to use in the next calculation.

$d \approx 427.68$

Building a Bridge
Step 2:

In right triangle OMP , use $\sin 31^\circ$ to find the length (b) of a bridge across the river.

$$\sin 31^\circ = \frac{b}{d}$$

$$\sin 31^\circ = \frac{b}{427.68}$$

$$427.68 \cdot \sin 31^\circ = b$$

$$220.28 \approx b$$

The length of a bridge will be approximately 220.28 yards.

Erecting a Steel Pole

A steel pole is to be erected on a hill with a slope of 8° .

The pole is 120 feet tall. Two guy-wires are to be anchored 80 feet from the base of the pole up and down the slope.

What are the lengths of the guy-wires?

Erecting a Steel Pole

Guy-wire

98°

120'

80'

80'

8°

Base line

Erecting a Steel Pole

Line is parallel to the base line forming equal alternate interior angles.

8°

80'

120'

80'

8°

Base line

Erecting a Steel Pole

90°

120'

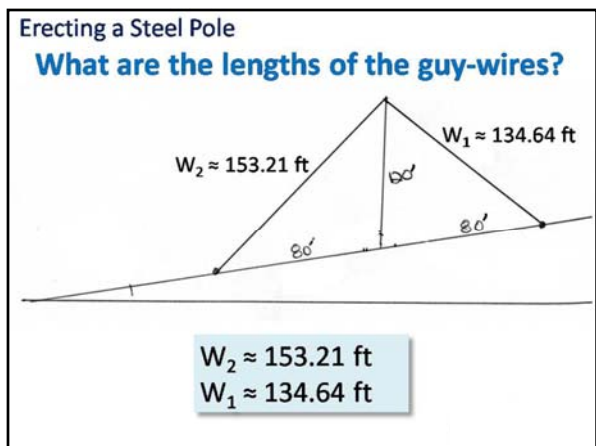
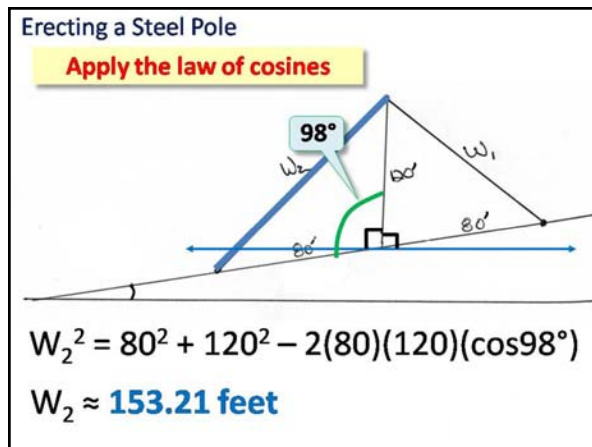
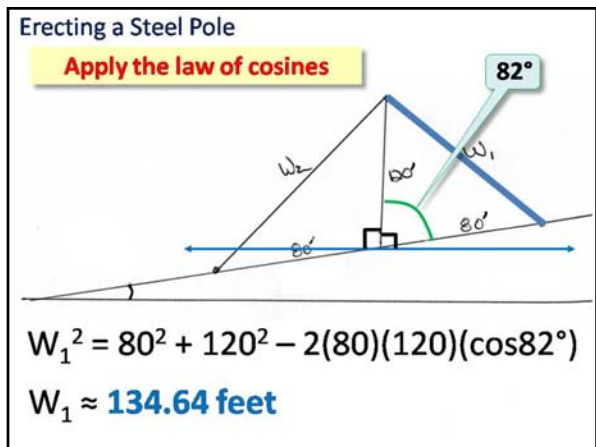
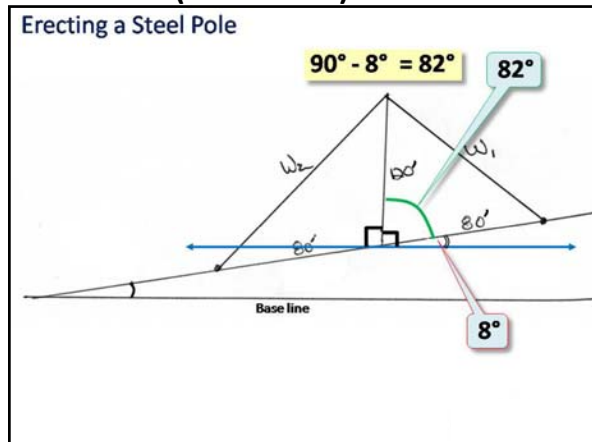
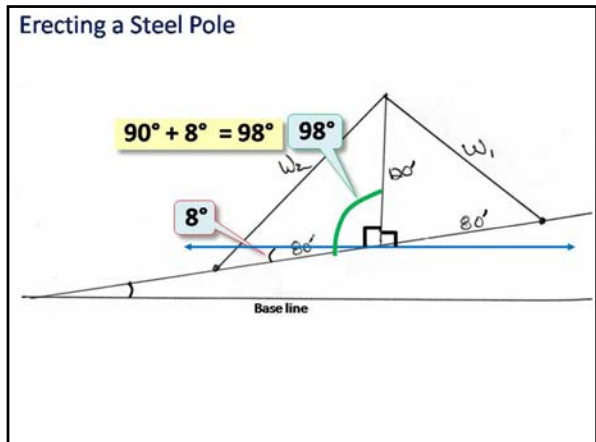
80'

80'

8°

Base line

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Summary

Use the laws of sines and cosines to find missing parts for any triangle

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