Slides from The Laws of Sines and Cosines (Tutorial 23)

The Laws of Sines and Cosines

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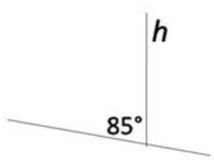
This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

Objectives

- **Learn to use the law of sines and the law of cosines**
- **We usually think of right triangles in trigonometry**
- **The law of sines and the law of cosines allow us to use trigonometry to find parts of any triangle**

Example 1 Law of Sines

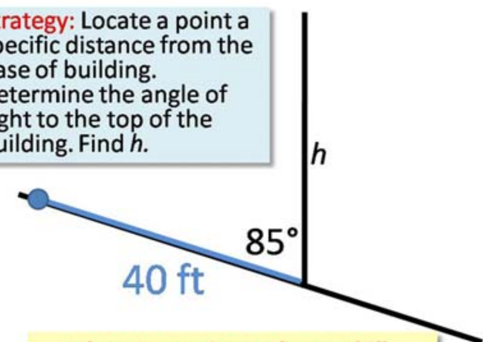
Find the height (*h*) of a building on a plot of land with a slight incline. The building makes an 85° angle with the ground.



The building is too tall to measure directly. Use indirect measurement.

Example 1 Law of Sines

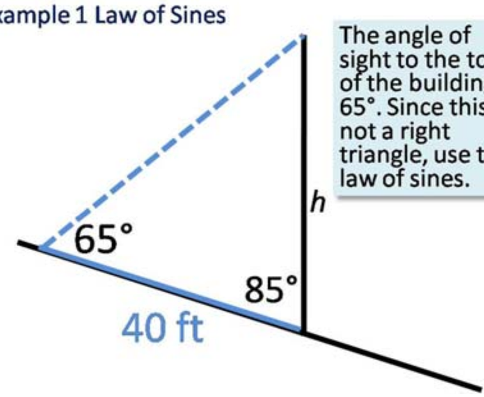
Strategy: Locate a point a specific distance from the base of building. Determine the angle of sight to the top of the building. Find *h*.



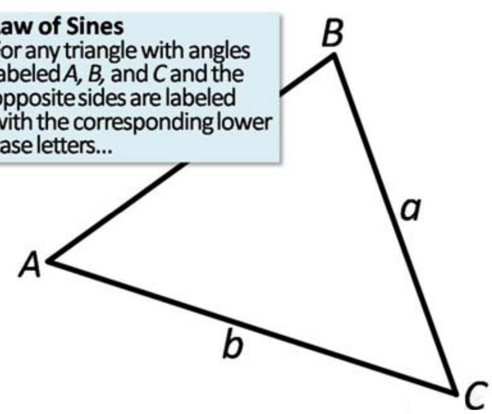
Choose a point 40 feet uphill.

Example 1 Law of Sines

The angle of sight to the top of the building is 65°. Since this is not a right triangle, use the law of sines.



Law of Sines
For any triangle with angles labeled *A, B,* and *C* and the opposite sides are labeled with the corresponding lower case letters…



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Law of Sines

For any triangle with angles labeled A, B, and C and the opposite sides labeled with the corresponding lower case letters, the following ratios are equal.

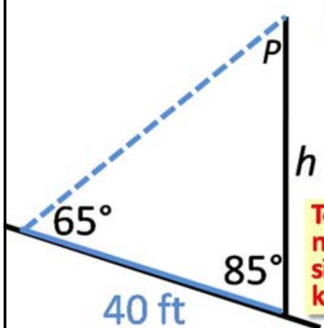
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Applying the Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- Use 2 ratios to find a missing part of a triangle
- Must know values for an angle and its opposite side along with the part opposite the missing part

Example 1 Law of Sines

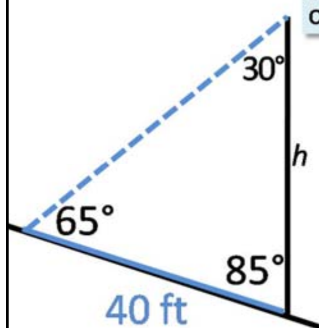


Find the height (h) of the building

$$\frac{\sin 65°}{h}$$

To get the second ratio needed for the law of sines, find angle P with known opposite side.

Example 1 Law of Sines



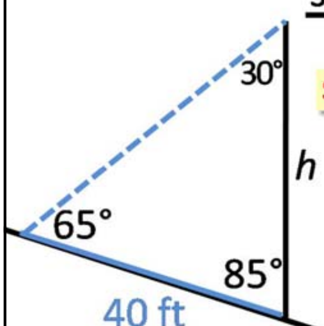
Sum of all three angles of a triangle is 180°

$$P + 65° + 85° = 180°$$

$$P = 180° - 65° - 85°$$

$$P = 30°$$

Example 1 Law of Sines



$$\frac{\sin 65°}{h} = \frac{\sin 30°}{40}$$

Solve proportion for h

Example 1 Law of Sines

$$\frac{\sin 65°}{h} = \frac{\sin 30°}{40}$$

Multiply both sides by h and then by 40

$$(\sin 65°)\, 40 = (\sin 30°)\, h$$

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Example 1 Law of Sines

$$\frac{\sin 65^\circ}{h} = \frac{\sin 30^\circ}{40}$$

Multiply both sides by *h* and then by 40

$$(\sin 65^\circ)\,40 = (\sin 30^\circ)\,h$$

Divide both sides by sin 30°

$$\frac{(\sin 65^\circ)\,40}{\sin 30^\circ} = \frac{(\sin 30^\circ)\,h}{\sin 30^\circ}$$

Example 1 Law of Sines

$$\frac{\sin 65^\circ}{h} = \frac{\sin 30^\circ}{40}$$

Multiply both sides by *h* and then by 40

$$(\sin 65^\circ)\,40 = (\sin 30^\circ)\,h$$

Divide both sides by sin 30°

$$\frac{(\sin 65^\circ)\,40}{\sin 30^\circ} = \frac{(\sin 30^\circ)\,h}{\sin 30^\circ}$$

Example 1 Law of Sines

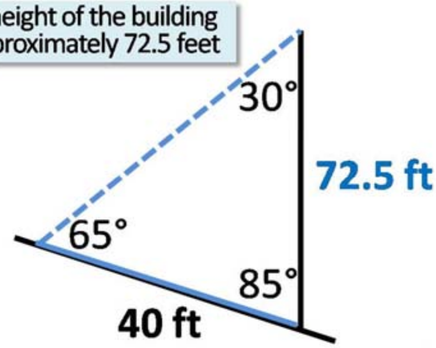
$$\frac{(\sin 65^\circ)\,40}{\sin 30^\circ} = h$$

Evaluate using your calculator.
Make sure calculator is in degree mode.

$$72.5 \approx h$$

Example 1 Law of Sines

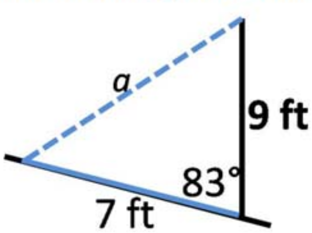
The height of the building is approximately 72.5 feet



Example 2 Law of Cosines

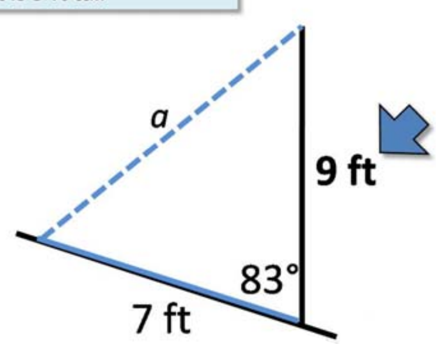
Install a support beam (*a*) to hold up a 9 foot sign post on a hill.

How long should the support beam be?



What we know:

- Sign post is 9 ft tall



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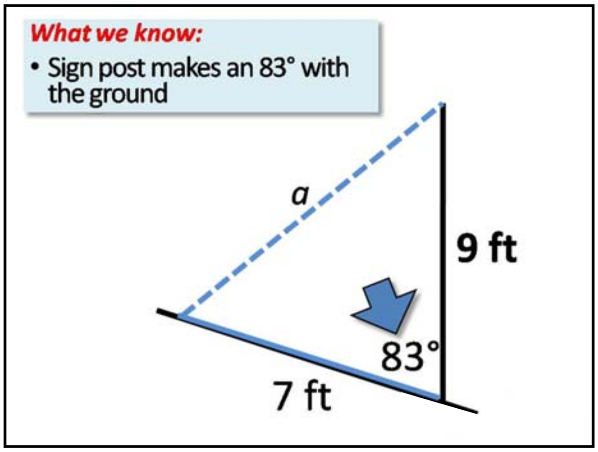

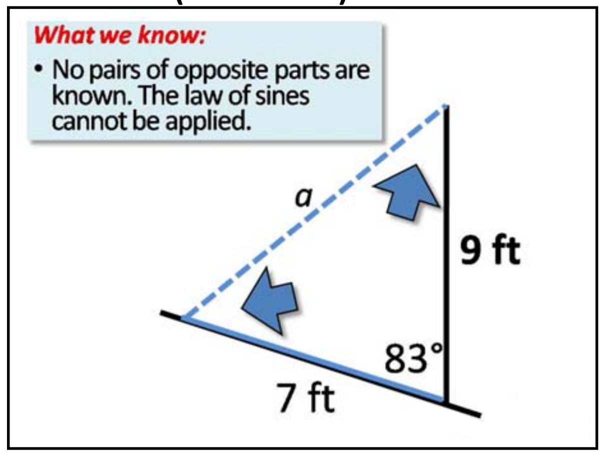

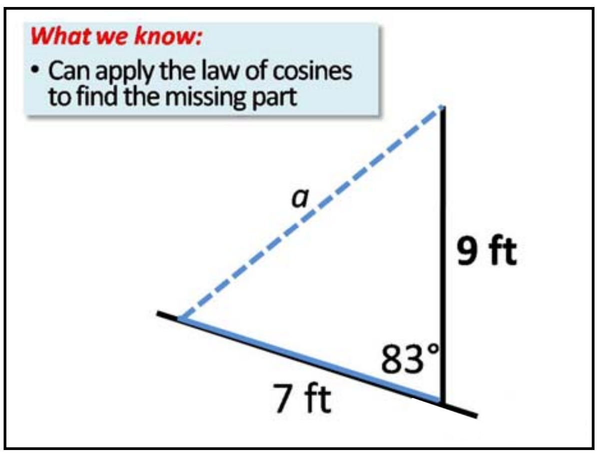

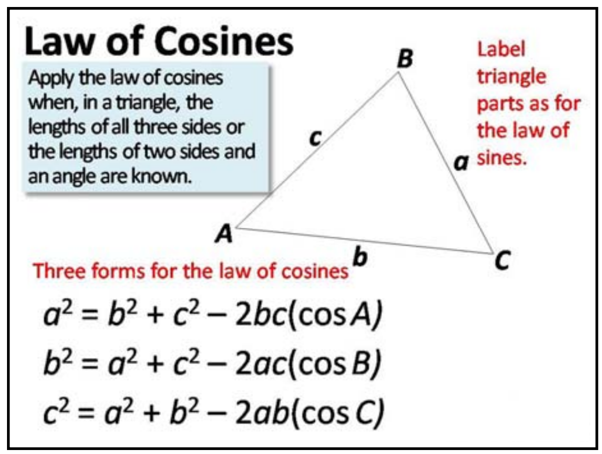

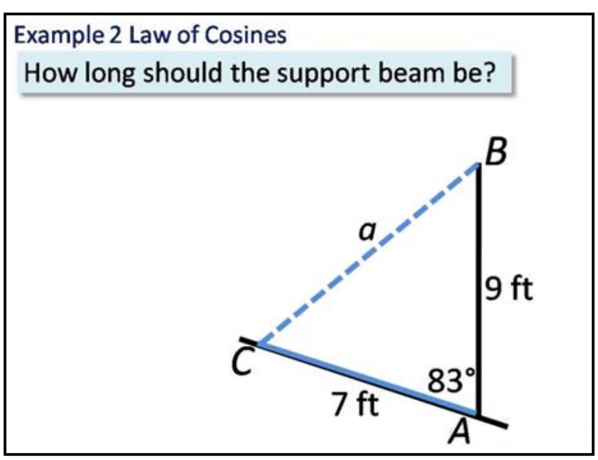

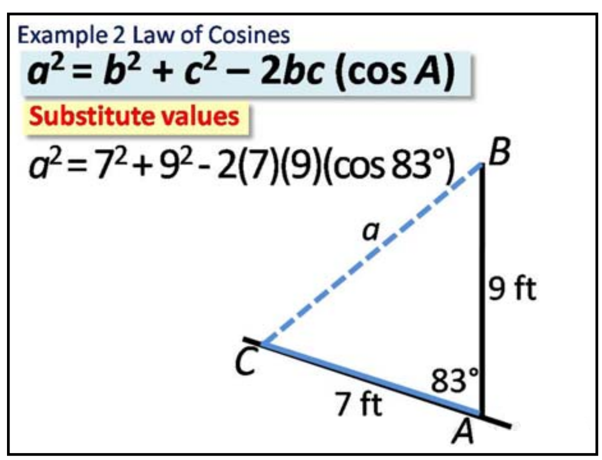

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Example 2 Law of Cosines

$a^2 = 7^2 + 9^2 - 2(7)(9)(\cos 83°)$

Multiply, evaluate cosine, and simplify

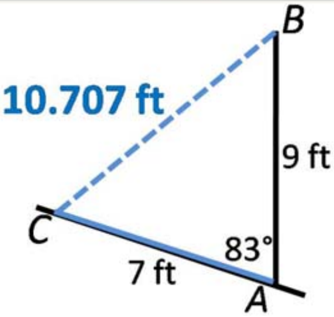
$a^2 = 49 + 81 - 15.356$

$a^2 \approx 114.644$

$a \approx 10.707$ ft

Example 2 Law of Cosines

Support beam must be approximately 10.707 ft



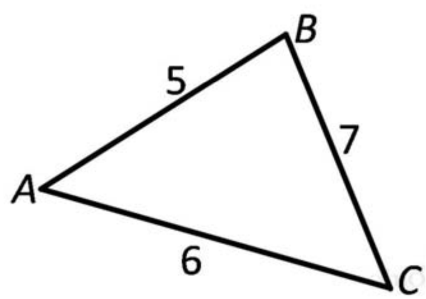
Solving Triangles

To solve a triangle:

- **Find all of the missing parts of the given triangle**
- **Use the law of sines, the law of cosines, and other mathematics formulas as needed.**

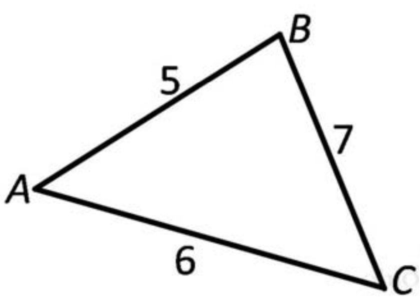
Example 3 Solving Triangles

Find the measures of angles *A*, *B*, and *C*



Example 3 Solving Triangles

Given the lengths of the three sides, apply **one** of the law of cosines formulas to find an angle.

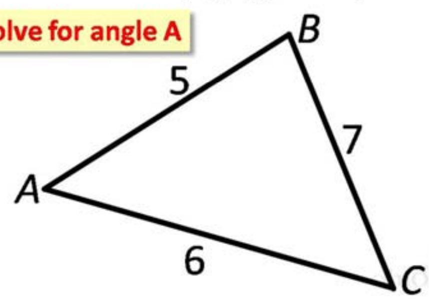


Example 3 Solving Triangles

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$7^2 = 5^2 + 6^2 - 2(5)(6)(\cos A)$

Solve for angle A



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Example 3 Solving Triangles

$$7^2 = 5^2 + 6^2 - 2(5)(6)(\cos A)$$

$$49 = 25 + 36 - 60(\cos A)$$

60 is the coefficient of the cosine function, so 36 minus 60 cannot be calculated

Example 3 Solving Triangles

$$7^2 = 5^2 + 6^2 - 2(5)(6)(\cos A)$$

$$49 = 25 + 36 - 60(\cos A)$$

$$49 = 61 - 60(\cos A)$$

$$-12 = -60(\cos A)$$

$$0.2 = \cos A$$

$\cos A$ is multiplied by -60, so undo by dividing both sides of the equation by -60.

Example 3 Solving Triangles

$$0.2 = \cos A$$

$$\cos^{-1}(0.2) \approx A$$

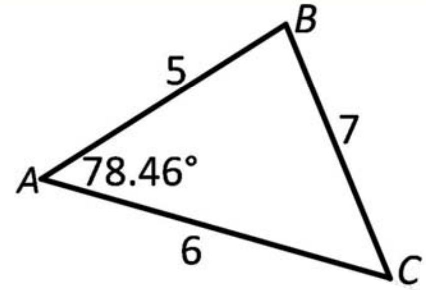
$$78.46° \approx A$$

To solve for A, find the inverse cosine. Use 2nd cos on your calculator.

Add to the diagram

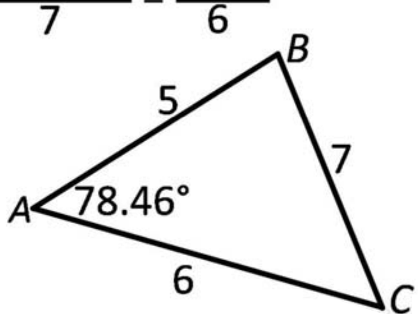
Example 3 Solving Triangles

Now, knowing one angle of the triangle, use the law of sines to find another angle.



Example 3 Solving Triangles

$$\frac{\sin 78.46°}{7} = \frac{\sin B}{6}$$



Example 3 Solving Triangles

$$\frac{\sin 78.46°}{7} = \frac{\sin B}{6}$$

Multiply both sides by 6

$$\frac{(\sin 78.46°)6}{7} = \sin B$$

$$.8398 \approx \sin B$$

$$57.12° \approx B$$

Use 2nd sin key to find B

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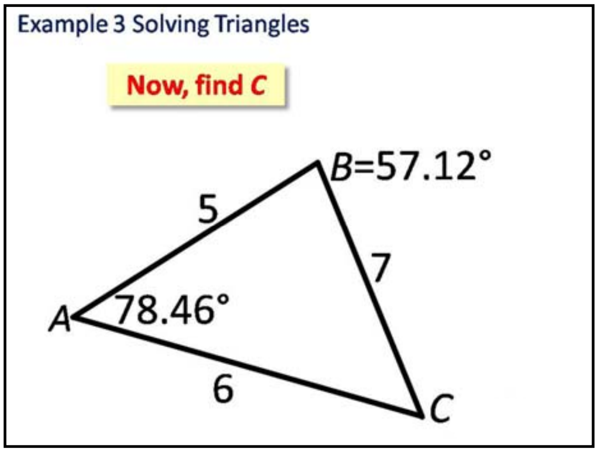
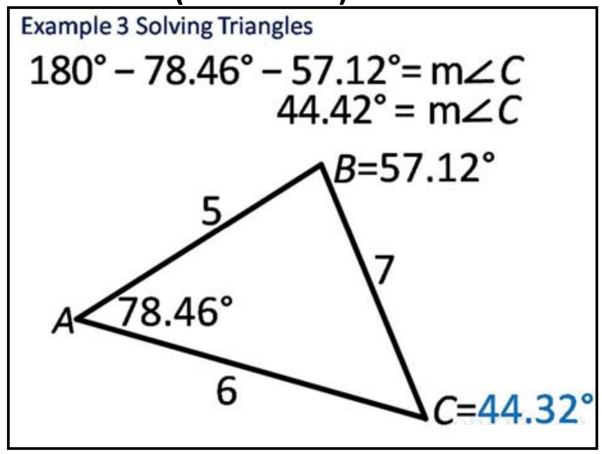
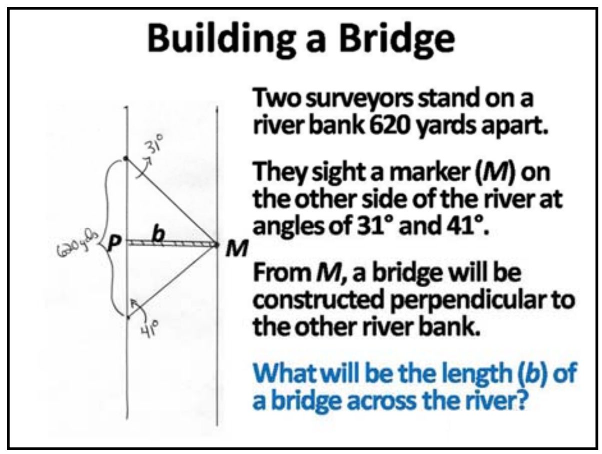
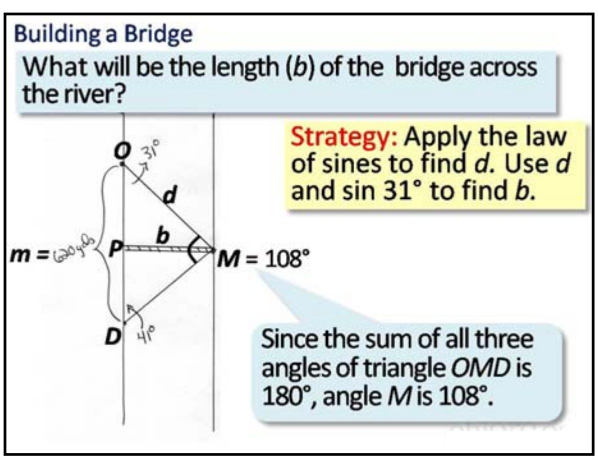
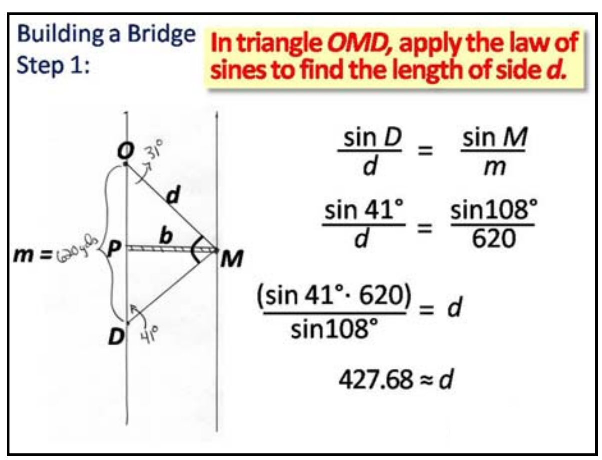




Two Application Problems







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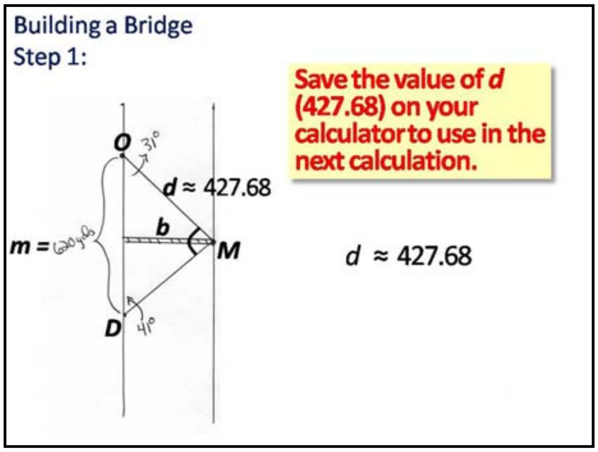
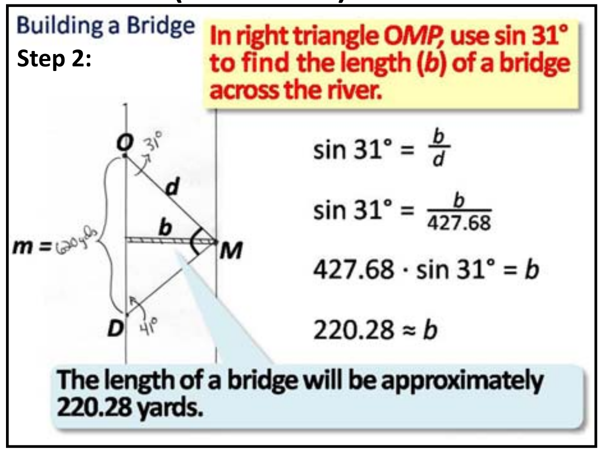


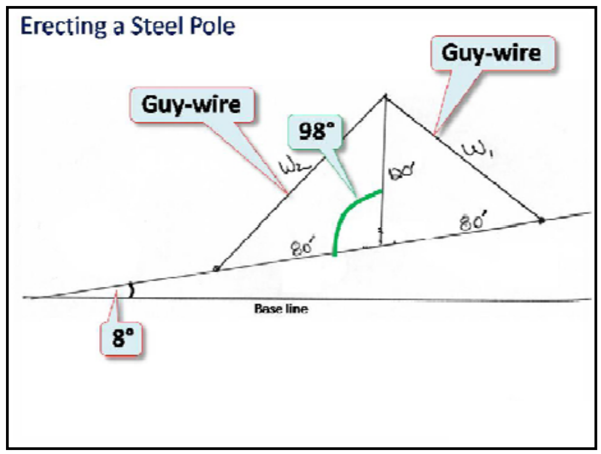


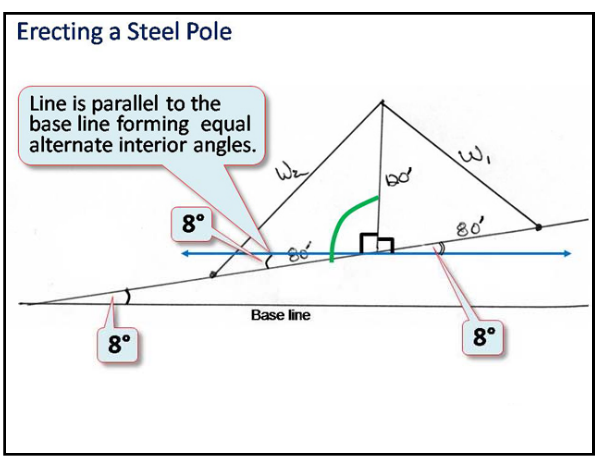
Erecting a Steel Pole

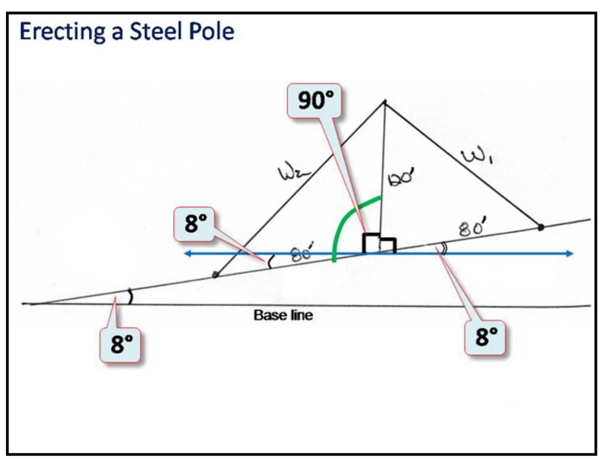
A steel pole is to be erected on a hill with a slope of 8°.

The pole is 120 feet tall. Two guy-wires are to be anchored 80 feet from the base of the pole up and down the slope.

What are the lengths of the guy-wires?







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