Self-Check 23
Laws of Sines and Cosines

1. In ΔXYZ, angle X measures 48.5°, side x measures 8.1 feet, and angle Z measures 25°. Apply the law of sines to find the length of side z. Include a sketch with your answer.

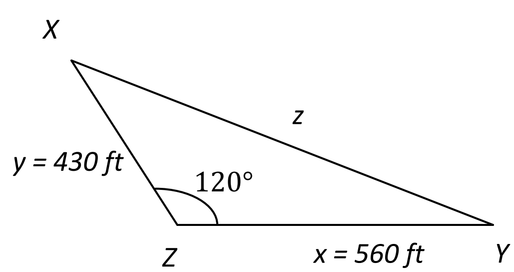
2. In ΔABC, angle C measures 125°, side b measures 5.3 cm, and side c measures 8.9 cm. Find all missing parts of the triangle. This is a two-step question that requires the use of the inverse sine. (*See guy-wire problem in tutorial #22 for calculator directions for finding an inverse trig function.*) Include a sketch with your answer.

3. Given ΔKLM with angle K equal to 141.43°, side l measuring 6.5 cm, and side m measuring 7.9 cm. Find the length of side k. Include a sketch with your answer.

4. In ΔABC, side a measures 8.2 cm, side b is 3.7 cm, and side c measures 10.8 cm. Find the measure of angle C. Don't forget that the largest angle of a triangle is opposite the longest side and the smallest angle is opposite the shortest side. (*See guy-wire problem in tutorial #22 for calculator directions for finding an inverse function.*) Include a sketch with your answer.

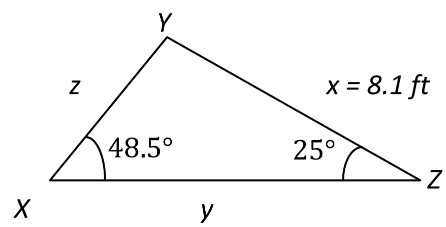
5. Using information and sketch from problem 4, find the measures of angles A and B.

6. Using the figure below, compare the cost of constructing a road on a rocky surface between points X and Y at the rate of \$30/ft with the cost of building the road on a soil surface with an indirect route from point X to point Z to point Y for \$20/ft. Is the shorter route is the most economical?



Solutions to Self-Check 23
Laws of Sines and Cosines

1. In Δ*XYZ*, angle *X* measures 48.5°, side *x* measures 8.1 feet, and angle *Z* measures 25°. Apply the law of sines to find the length of side *z*.



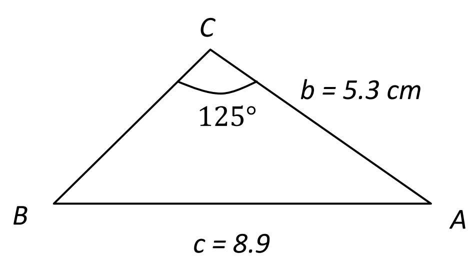
$$\frac{\sin 48.5°}{8.1} = \frac{\sin 25°}{z}$$

$$z \cdot \sin 48.5° = 8.1 \cdot \sin 25°$$

$$z = \frac{8.1 \cdot \sin 25°}{\sin 48.5°}$$

$$z \approx 4.570641261 \ or \ z \approx 4.5 \ feet$$

2. In Δ*ABC*, angle *C* measures 125°, side *b* measures 5.3 cm, and side *c* measures 8.9 cm. Find all missing parts of the triangle. This is a two-step question that requires the use of the inverse sine. (*See guy-wire problem in tutorial #22 for calculator directions for finding an inverse trig function.*)



$$\frac{\sin 125°}{8.9} = \frac{\sin B}{5.3}$$

$$\frac{5.3 \cdot \sin 125°}{8.9} = \sin B$$

$$.4878096444 \approx \sin B$$

$$\sin^{-1}(.4878096444) \approx B$$

$$29.19671722 \approx B \text{ or } m\angle B \approx 29.2°$$

Since the sum of angles *A*, *B*, and *C* is 180°, angle *A* is approximately 25.8°. Apply the law of sines a second time.

$$\frac{\sin 25.8°}{a} = \frac{\sin 125°}{8.9}$$

$$\frac{8.9 \sin 25.8°}{\sin 125°} = a$$

$$4.728739691 \approx a \text{ or measure of side } a \text{ is approximately 4.73 inches}$$

Solutions to Self-Check 23
Laws of Sines and Cosines

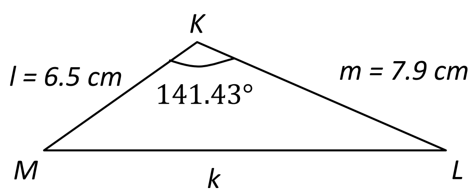
3. Given Δ*KLM* with angle *K* equal to 141.43°, side *l* measuring 6.5 cm, and side *m* measuring 7.9 cm. Find the length of side *k*. Apply the law of cosines.

$$k^2 = l^2 + m^2 - 2(l)(m)\cos K$$

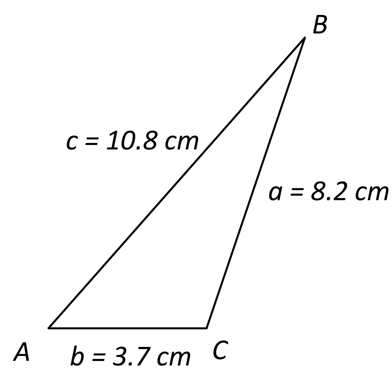
$$k^2 = 6.5^2 + 7.9^2 - 2(6.5)(7.9)\cos 141.43°$$

$$k^2 \approx 184.8956898$$

$k \approx 13.59763545$ or measure of side k is approximately 13.6 feet



4. In Δ*ABC,* side *a* measures 8.2 cm, side *b* is 3.7 cm, and side *c* measures 10.8 cm. Find the measure of angle *C*. Don't forget that the largest angle of a triangle is opposite the longest side and the smallest angle is opposite the shortest side. (*See guy-wire problem in tutorial #22 for calculator directions for finding an inverse trig function.*)



$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

$$10.8^2 = 8.2^2 + 3.7^2 - 2(8.2)\cdot(3.7)\cdot\cos C$$

$$116.64 = 67.24 + 13.69 - 60.68\cos C$$

$$35.71 = -60.68\cos C$$

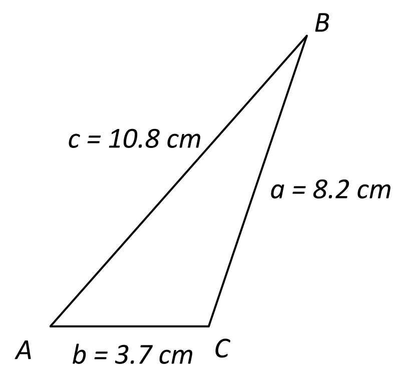
$$-.5884970336 = \cos C$$

$$cos^{-1}(-.5884970336) = C$$

$$126.0504255 \approx C \text{ or } m\angle C \approx 126.1°$$

Solutions to Self-Check 23
Laws of Sines and Cosines

5. Using information, solution, and sketch from problem 4, find the measures of angles *A* and *B*. When given three sides of a triangle, first use the law of cosines to find the measure of the largest angle (problem 4). Next, use the law of sines to find the measure of a second angle. Finally, knowing that the sum of angles *A*, *B*, and *C* is 180°, calculate the measure of the third angle.



$$\frac{\sin A}{a} = \frac{\sin 126.1°}{c}$$

$$\frac{\sin A}{a} \approx \frac{.80799}{10.8}$$

$$\sin A \approx .074813878 \cdot 8.2$$

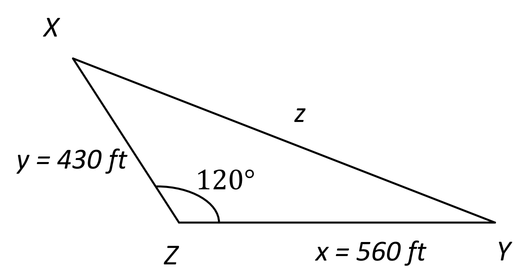
$$\sin A \approx .61347$$

$$A \approx \sin^{-1}(.61347)$$

$$\text{m}\angle A \approx 37.8°$$

$$\text{m}\angle B \approx 180° - 126.1° - 37.8° \text{ or } \text{m}\angle B \approx 16.1°$$

6. Using the figure below, compare the cost of constructing a road on a rocky surface between points *X* and *Y* at the rate of $30/ft with the cost of building the road on a soil surface with an indirect route from point *X* to point *Z* to point *Y* for $20/ft. Is the shorter route is the most cost effective?



$$z^2 = x^2 + y^2 - (x)(y)\cos Z$$

$$z^2 = 560^2 + 430^2 - 2(560)(430)\cos 120°$$

$$z^2 \approx 739300$$

$z \approx 859.8255673$ or measure of side z is approximately 859.83ft

Approximate cost for direct stony surface route ($30/ft) is $25,794 and cost for the indirect soil surface route ($20/ft) is $19,860. Therefore, the longer, indirect route is most cost effective.