Slides from Probability Basics (Tutorial 24)

Exploring Theoretical and Experimental Probability

Probability is a measure of the likelihood of an event or outcome occurring.

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This handout contains selected slides to use when reviewing this tutorial topic with or without the video. To access all slides, open thumbnail link on the tutorial interface.

The *probability* of an event or outcome is a number 0 between 1.

Probabilities can be written as a fraction, a decimal, or a percent.

For example, the probability of getting "heads" when tossing a coin is

$\frac{1}{2}$ or 0.5 or 50%

An **impossible event** has a probability of 0.

An **event that is certain** has a probability of 1.

A probability cannot be greater than 1 (or 100%) because an event can't be more likely than certain!

Theoretical Probability

List all the possible outcomes for an event, called the **sample space**.

Count the number of possible outcomes that match the outcome for which you are finding the probability.

This assumes all of the possible outcomes are equally-likely to happen.

Theoretical Probability

The ratio of favorable or desired outcomes to all possible equally-likely outcomes.

$$P(\text{desired outcome}) = \frac{\text{number of ways desired outcome can occur}}{\text{number of all possible equally-likely outcomes}}$$

Experimental Probability

A ratio based on data from repeated trials of an event gathered through an investigation or simulation.

$$P(\text{desired outcome}) = \frac{\text{number of ways desired outcome can occur}}{\text{number of trials}}$$

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Finding Theoretical Probability

$$P(\text{desired outcome}) = \frac{\text{number of ways desired outcome can occur}}{\text{number of all possible equally-likely outcomes}}$$

To find a theoretical probability, one must find the number of all possible outcomes.

It is often helpful to make an organized list or tree-diagram to find all possible outcomes.

Finding Theoretical Probability

$$P(\text{2 boys \& 2 girls}) = \frac{\text{number with 2 boys and 2 girls}}{\text{number of possible arrangements of 4 children}}$$

What is the probability of a family with four children having two boys and two girls?

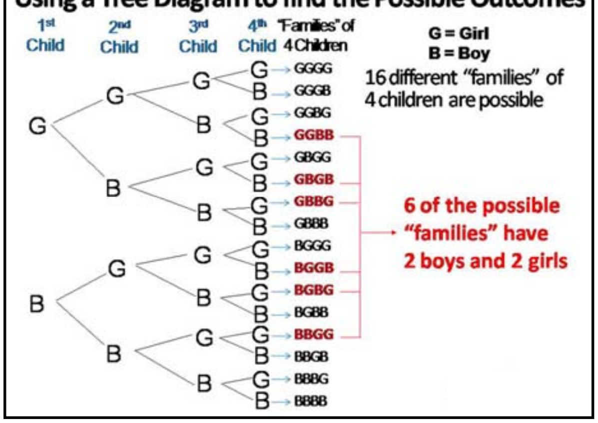
Finding Theoretical Probability

$$P(\text{2 boys \& 2 girls}) = \frac{\text{number with 2 boys and 2 girls}}{\text{number of possible arrangements of 4 children}}$$

It is assumed the birth of each child is an independent event.

Two (or more) events are independent when the outcome of the first event does not affect the outcome of the second event.

Using a Tree Diagram to find the Possible Outcomes



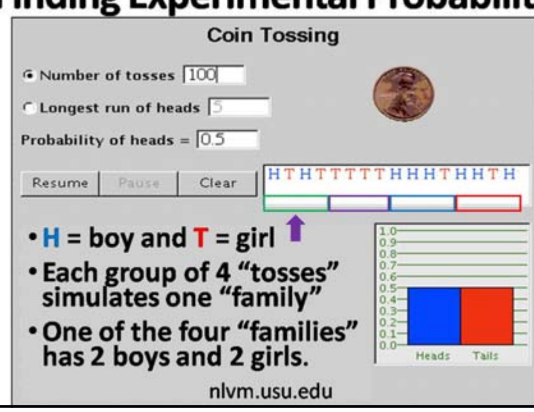
Finding Theoretical Probability

$$P(\text{2 boys \& 2 girls}) = \frac{\text{number with 2 boys and 2 girls}}{\text{number of possible arrangements of 4 children}}$$

$$P(\text{2 boys \& 2 girls}) = \frac{\text{6 "families" with 2 boys and 2 girls}}{\text{16 possible "families" of 4 children}}$$

$$P(\text{2 boys \& 2 girls}) = \frac{6}{16} = \frac{3}{8} = 0.375 = \mathbf{37.5\%}$$

Finding Experimental Probability



- H = boy and T = girl
- Each group of 4 "tosses" simulates one "family"
- One of the four "families" has 2 boys and 2 girls.

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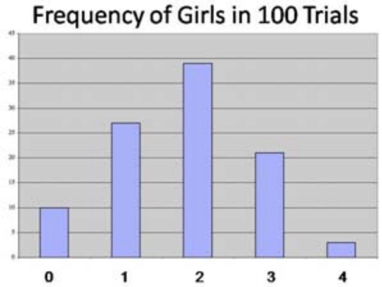
Results of 100 trials (400 tosses)

Girls	Freq
0	10
1	27
2	39
3	21
4	3

Frequency of Girls in 100 Trials



Finding Experimental Probability

$$P(2 \text{ boys } \& \ 2 \text{ girls}) = \frac{\text{number with trials with 2 boys and 2 girls}}{\text{number of trials}}$$

$$P(2 \text{ boys } \& \ 2 \text{ girls}) = \frac{\text{39 trials with 2 boys and 2 girls}}{\text{100 trials}}$$

$$P(2 \text{ boys } \& \ 2 \text{ girls}) = \frac{39}{100} = 0.39 = \mathbf{39\%}$$

Experimental Probability

- Simulations may be used to model and estimate the probability of an event.
- Many trials are needed to be confident in the accuracy of the experimental probability.

- The gender of a child at birth is a random event that is approximately 50% male and 50% female.
- The fact that a family has two boys in a row does not mean the family is "due" to have a girl as a third child.
- Each trial (or birth) has the same probability – approximately 50% male and 50% female.

Fundamental Counting Principle

If one event can happen in ***a*** ways and a second independent event can happen in ***b*** ways, the two can occur together in ***a* • *b*** ways.

Fundamental Counting Principle

Find the number of possible outcomes for "families" of four children:

2 • 2 • 2 • 2 = 16 possible outcomes ("families" of 4 children)

1st child, 2nd child, 3rd child, 4th child

Two possible outcomes for each child: boy or girl

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Fundamental Counting Principle

How many different Ohio license plates with three letters followed by four single digits can be made?

26 • 26 • 26 • 10 • 10 • 10 • 10 = 175,760,000 license plates

1st letter, 2nd letter, 3rd letter, 1st digit, 2nd digit, 3rd digit, 4th digit

26 possible outcomes for each letter
10 possible outcomes for each digit

Finding Theoretical Probability

What is the probability of having an exact match on the Ohio Pick Three game?

10 • 10 • 10 = 1000 possible Pick Three numbers

1st digit, 2nd digit, 3rd digit

Finding Theoretical Probability

What is the probability of having an exact match on the Ohio Pick Three game?

$$\text{P(exact match)} = \frac{\text{number of ways desired outcome can occur}}{\text{number of all possible Pick Three numbers}}$$

$$\text{P(exact match)} = \frac{1}{1000} = 0.001 = \mathbf{0.1\%}$$

Finding Theoretical Probability

What is the probability of winning if you picked 7, 8 and 5 and "boxed" your numbers?

$$\text{P(winning)} = \frac{\text{number of 3-digit numbers with a 7, 8 and 5}}{\text{number of all possible Pick Three numbers}}$$

Finding Theoretical Probability

Find the number of three-digit numbers with a 7, 8, and 5

3 • 2 • 1 = 6 three-digit numbers with a 7, 8, and 5

Possible 1st digits (7, 8, or 5)
Possible 2nd digits (two remaining digits)
Possible 3rd digits (one remaining digit)

Finding Theoretical Probability

What is the probability of winning if you picked 7, 8 and 5 and "boxed" your numbers?

$$\text{P(winning)} = \frac{\text{number of 3-digit numbers with a 7, 8, and 5}}{\text{number of all possible Pick Three numbers}}$$

$$\text{P(winning)} = \frac{6}{1000} = 0.006 = \mathbf{0.6\%}$$

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What if all the events in a sample space are not equally likely?

Finding Theoretical Probability

A *Punnett Square* is a simple graphical way of finding the potential outcomes for a trait from two parents.

genetic contribution of one parent

genetic contribution of the other parent

	B	O
B	BB	BO
O	OB	OO

Offspring's possible genotypes

The science is simplified for this problem.

Blood Type Example

genetic contribution of one parent

genetic contribution of the other parent

	B	O
B	BB	BO
O	OB	OO

Offspring's possible genotypes

Legend: Type B blood = yellow; Type O blood = pink

$$P(\text{type B blood}) = \frac{\text{number of genotypes resulting in type B}}{\text{number of all possible genotypes}} = \frac{3}{4} = 75\%$$

$$P(\text{type O blood}) = \frac{\text{number of genotypes resulting in type O}}{\text{number of all possible genotypes}} = \frac{1}{4} = 25\%$$

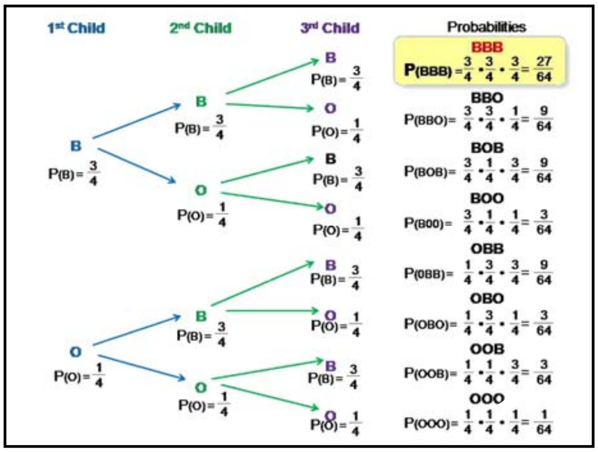
Finding Theoretical Probability

Two parents, both with genotype BO blood, have three children.

What is the probability that all three children have Type B blood?

$$\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64} \approx 42\ \%$$

probability of 1st child having type B; probability of 2nd child having type B; probability of 3rd child having type B



If the probability of an outcome (A) of an independent event is

$$P(A) = r$$

then the probability of having A occur n times in a row is:

$$P(A \text{ occurs } n \text{ times in a row}) = r^n$$

For example, the probability of having A occur ten times in a row is:

$$P(A \text{ occurs 10 times in a row}) = r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r = r^{10}$$

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Summary

- Reviewed theoretical probability and experimental probability
- Made simple counting arguments
- Found the probability of independent events